

twinstim: An endemic-epidemic modeling framework for spatio-temporal point patterns

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Abstract

The availability of geocoded health data and the inherent temporal structure of communicable diseases have led to an increased interest in statistical models and software for spatio-temporal data with epidemic features. The R package **surveillance** can handle various levels of aggregation at which infective events have been recorded. This vignette illustrates the analysis of *point-referenced* surveillance data using the endemic-epidemic point process model “`twinstim`” proposed by Meyer, Elias, and Höhle (2012) and extended in Meyer and Held (2014). We first describe the general modeling approach and then exemplify data handling, model fitting, visualization, and simulation methods for time-stamped geo-referenced case reports of invasive meningococcal disease (IMD) caused by the two most common bacterial finetypes of meningococci in Germany, 2002–2008.

Keywords: spatio-temporal point pattern, endemic-epidemic modeling, infectious disease epidemiology, self-exciting point process, spatial interaction function, branching process with immigration.

1. Model class: `twinstim`

Infective events occur at specific points in continuous space and time, which gives rise to a spatio-temporal point pattern $\{(\mathbf{s}_i, t_i) : i = 1, \dots, n\}$ from a region \mathbf{W} observed during a period $(0, T]$. The locations \mathbf{s}_i and time points t_i of the n events can be regarded as a realization of a self-exciting spatio-temporal point process, which can be characterized by its conditional intensity function (CIF, also termed intensity process) $\lambda(\mathbf{s}, t)$. It represents the instantaneous event rate at location \mathbf{s} at time point t given all past events, and is often more verbosely denoted by λ^* or by explicit conditioning on the “history” \mathcal{H}_t of the process. Daley and Vere-Jones (2003, Chapter 7) provide a rigorous mathematical definition of this concept, which is key to likelihood analysis and simulation of “evolutionary” point processes.

Meyer *et al.* (2012) formulated the model class “`twinstim`” – a *two*-component spatio-temporal intensity model – by a superposition of an endemic and an epidemic component:

$$\lambda(\mathbf{s}, t) = \nu_{[\mathbf{s}][t]} + \sum_{j \in I(\mathbf{s}, t)} \eta_j f(\|\mathbf{s} - \mathbf{s}_j\|) g(t - t_j). \quad (1)$$

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This model constitutes a branching process with immigration. Part of the event rate is due to the first, endemic component, which reflects sporadic events caused by unobserved sources of infection. This background rate of new events is modeled by a log-linear predictor $\nu_{[\mathbf{s}][t]}$ incorporating regional and/or time-varying characteristics. Here, the space-time index $[\mathbf{s}][t]$ refers to the region covering \mathbf{s} during the period containing t and thus spans a whole spatio-temporal grid on which the involved covariates are measured, e.g., district \times month. We will later see that the endemic component therefore simply equals an inhomogeneous Poisson process for the event counts by cell of that grid.

The second, observation-driven epidemic component adds “infection pressure” from the set

$$I(\mathbf{s}, t) = \{j : t_j < t \wedge t - t_j \leq \tau_j \wedge \|\mathbf{s} - \mathbf{s}_j\| \leq \delta_j\}$$

of past events and hence makes the process “self-exciting”. During its infectious period of length τ_j and within its spatial interaction radius δ_j , the model assumes each event j to trigger further events, which are called offspring, secondary cases, or aftershocks, depending on the application. The triggering rate (or force of infection) is proportional to a log-linear predictor η_j associated with event-specific characteristics (“marks”) \mathbf{m}_j , which are usually attached to the point pattern of events. The decay of infection pressure with increasing spatial and temporal distance from the infective event is modeled by parametric interaction functions f and g , respectively. A simple assumption for the time course of infectivity is $g(t) = 1$. Alternatives include exponential decay, a step function, or empirically derived functions such as Omori’s law for aftershock intervals. With regard to spatial interaction, a Gaussian kernel $f(x) = \exp\{-x^2/(2\sigma^2)\}$ could be chosen. However, in modeling the spread of human infectious diseases on larger scales, a heavy-tailed power-law kernel $f(x) = (x + \sigma)^{-d}$ was found to perform better (Meyer and Held 2014). The (possibly infinite) upper bounds τ_j and δ_j provide a way of modeling event-specific interaction ranges. However, since these need to be pre-specified, a common assumption is $\tau_j \equiv \tau$ and $\delta_j \equiv \delta$, where the infectious period τ and the spatial interaction radius δ are determined by subject-matter considerations.

1.1. Model-based effective reproduction numbers

Similar to the simple SIR model (see, e.g., Keeling and Rohani 2008, Section 2.1), the above point process model (1) features a reproduction number derived from its branching process interpretation. As soon as an event occurs (individual becomes infected), it triggers offspring (secondary cases) around its origin (\mathbf{s}_j, t_j) according to an inhomogeneous Poisson process with rate $\eta_j f(\|\mathbf{s} - \mathbf{s}_j\|) g(t - t_j)$. Since this triggering process is independent of the event’s parentage and of other events, the expected number μ_j of events triggered by event j can be obtained by integrating the triggering rate over the observed interaction domain:

$$\mu_j = \eta_j \cdot \left[\int_0^{\min(T-t_j, \tau_j)} g(t) dt \right] \cdot \left[\int_{\mathbf{R}_j} f(\|\mathbf{s}\|) d\mathbf{s} \right], \quad (2)$$

where

$$\mathbf{R}_j = (b(\mathbf{s}_j, \delta_j) \cap \mathbf{W}) - \mathbf{s}_j \quad (3)$$

is event j ’s influence region centered at \mathbf{s}_j , and $b(\mathbf{s}_j, \delta_j)$ denotes the disc centered at \mathbf{s}_j with radius δ_j . Note that the above model-based reproduction number μ_j is event-specific since it depends on event marks through η_j , on the interaction ranges δ_j and τ_j , as well as on the

event location \mathbf{s}_j and time point t_j . If the model assumes unique interaction ranges δ and τ , a single reference number of secondary cases can be extrapolated from Equation 2 by imputing an unbounded domain $\mathbf{W} = \mathbb{R}^2$ and $T = \infty$ (Meyer, Warnke, Rössler, and Held 2016).

Equation 2 can also be motivated by looking at a spatio-temporal version of the simple SIR model wrapped into the `twinstim` class (1). This means: no endemic component, homogeneous force of infection ($\eta_j \equiv \beta$), homogeneous mixing in space ($f(x) = 1$, $\delta_j \equiv \infty$), and exponential decay of infectivity ($g(t) = e^{-\alpha t}$, $\tau_j \equiv \infty$). Then, for $T \rightarrow \infty$,

$$\mu = \beta \cdot \left[\int_0^\infty e^{-\alpha t} dt \right] \cdot \left[\int_{\mathbf{W}-\mathbf{s}_j} 1 d\mathbf{s} \right] = \beta \cdot |\mathbf{W}|/\alpha,$$

which corresponds to the basic reproduction number known from the simple SIR model by interpreting $|\mathbf{W}|$ as the population size, β as the transmission rate and α as the removal rate. If $\mu < 1$, the process is sub-critical, i.e., its eventual extinction is almost sure.

However, it is crucial to understand that in a full model with an endemic component, new infections may always occur via “immigration”. Hence, reproduction numbers in `twinstim` are adjusted for infections occurring independently of previous infections. This also means that a misspecified endemic component may distort model-based reproduction numbers (Meyer *et al.* 2016). Furthermore, under-reporting and implemented control measures imply that the estimates are to be thought of as *effective* reproduction numbers.

1.2. Likelihood inference

The log-likelihood of the point process model (1) is a function of all parameters in the log-linear predictors $\nu_{[\mathbf{s}][t]}$ and η_j and in the interaction functions f and g . It has the form

$$\left[\sum_{i=1}^n \log \lambda(\mathbf{s}_i, t_i) \right] - \int_0^T \int_{\mathbf{W}} \lambda(\mathbf{s}, t) d\mathbf{s} dt. \quad (4)$$

To estimate the model parameters, we maximize the above log-likelihood numerically using the quasi-Newton algorithm available through the R function `nlminb`. We thereby employ the analytical score function and an approximation of the expected Fisher information worked out by Meyer *et al.* (2012, Web Appendices A and B).

The space-time integral in the log-likelihood (4) poses no difficulties for the endemic component of $\lambda(\mathbf{s}, t)$, since $\nu_{[\mathbf{s}][t]}$ is defined on a spatio-temporal grid. However, integration of the epidemic component involves two-dimensional integrals $\int_{\mathbf{R}_i} f(\|\mathbf{s}\|) d\mathbf{s}$ over the influence regions \mathbf{R}_i , which are represented by polygons (as is \mathbf{W}). Similar integrals appear in the score function, where $f(\|\mathbf{s}\|)$ is replaced by partial derivatives with respect to kernel parameters. Calculation of these integrals is trivial for (piecewise) constant f , but otherwise requires numerical integration. For this purpose, the R package **polyCub** (Meyer 2016) offers cubature methods for polygonal domains as described in Meyer and Held (2014, Supplement B, Section 2). For Gaussian f , we apply a midpoint rule with σ -adaptive bandwidth and use product Gauss cubature (Sommariva and Vianello 2007) to approximate the integrals in the score function. For the recently implemented power-law kernels, we established a cubature method which takes advantage of the assumed isotropy of spatial interaction such that numerical integration remains in only one dimension (Meyer and Held 2014, Supplement B, Section 2.4). We **memoise** (Wickham, Hester, Müller, and Cook 2017) the cubature function during log-likelihood maximization to avoid integration for unchanged parameters of f .

1.3. Special cases: Single-component models

If the *epidemic* component is omitted in Equation 1, the point process model becomes equivalent to a Poisson regression model for aggregated counts. This provides a link to ecological regression approaches in general and to the count data model `hhh4` illustrated in `vignette("hhh4")` and `vignette("hhh4_spacetime")`. To see this, recall that the endemic component $\nu_{[s][t]}$ is piecewise constant on the spatio-temporal grid with cells $([s], [t])$. Hence the log-likelihood (4) of an endemic-only `twinstim` simplifies to a sum over all these cells,

$$\sum_{[s],[t]} \left\{ Y_{[s][t]} \log \nu_{[s][t]} - |[s]| |[t]| \nu_{[s][t]} \right\},$$

where $Y_{[s][t]}$ is the aggregated number of events observed in cell $([s], [t])$, and $|[s]|$ and $|[t]|$ denote cell area and length, respectively. Except for an additive constant, the above log-likelihood is equivalently obtained from the Poisson model $Y_{[s][t]} \sim \text{Po}(|[s]| |[t]| \nu_{[s][t]})$. This relation offers a means of code validation using the established `glm` function to fit an endemic-only `twinstim` model – see the examples in `help("glm_epidataCS")`.

If, in contrast, the *endemic* component is omitted, all events are necessarily triggered by other observed events. For such a model to be identifiable, a prehistory of events must exist to trigger the first event, and interaction typically needs to be unbounded such that each event can actually be linked to potential source events.

1.4. Extension: `twinstim` with event types

To model the example data on invasive meningococcal disease in the remainder of this section, we actually need to use an extended version $\lambda(\mathbf{s}, t, k)$ of Equation 1, which accounts for different event types k with own transmission dynamics. This introduces a further dimension in the point process, and the second log-likelihood component in Equation 4 accordingly splits into a sum over all event types. We refer to Meyer *et al.* (2012, Sections 2.4 and 3) for the technical details of this type-specific `twinstim` class. The basic idea is that the meningococcal finetypes share the same endemic pattern (e.g., seasonality), while infections of different finetypes are not associated via transmission. This means that the force of infection is restricted to previously infected individuals with the same bacterial finetype k , i.e., the epidemic sum in Equation 1 is over the set $I(\mathbf{s}, t, k) = I(\mathbf{s}, t) \cap \{j : k_j = k\}$. The implementation has limited support for type-dependent interaction functions f_{k_j} and g_{k_j} (not further considered here).

2. Data structure: `epidataCS`

The first step toward fitting a `twinstim` is to turn the relevant data into an object of the dedicated class `epidataCS`.¹ The primary ingredients of this class are a spatio-temporal point pattern (`events`) and its underlying observation region (`W`). An additional spatio-temporal grid (`stgrid`) holds (time-varying) area-level covariates for the endemic regression part. We exemplify this data class by the `epidataCS` object for the 636 cases of invasive meningococcal disease in Germany originally analyzed by Meyer *et al.* (2012). It is already contained in the `surveillance` package as `data("imdepi")` and has been constructed as follows:

¹ The suffix “CS” indicates that the data-generating point process is indexed in continuous space.

```
R> imdepi <- as.epidataCS(events = events, W = stateD, stgrid = stgrid,
+   qmatrix = diag(2), nCircle2Poly = 16)
```

The function `as.epidataCS` checks the consistency of the three data ingredients described in detail below. It also pre-computes auxiliary variables for model fitting, e.g., the individual influence regions (3), which are intersections of the observation region with discs approximated by polygons with `nCircle2Poly = 16` edges. The intersections are computed using functionality of the package **polyclip** (Johnson and Baddeley 2017). For multitype epidemics as in our example, the additional indicator matrix `qmatrix` specifies transmissibility across event types. An identity matrix corresponds to an independent spread of the event types, i.e., cases of one type can not produce cases of another type.

2.1. Data ingredients

The core `events` data must be provided in the form of a `SpatialPointsDataFrame` as defined by the package **sp** (Pebesma and Bivand 2016):

```
R> summary(events)
```

Object of class `SpatialPointsDataFrame`

Coordinates:

min max

x 4039 4665

y 2710 3525

Is projected: TRUE

proj4string :

[+init=epsg:3035 +units=km +proj=laea +lat_0=52 +lon_0=10 +x_0=4321000 +y_0=3210000

+ellps=GRS80 +no_defs]

Number of points: 636

Data attributes:

	time	tile	type	eps.t	eps.s	sex	agegrp
Min.	: 0.2	05354	: 34	B:336	Min. :30	Min. :200	female:292 [0,3) :194
1st Qu.:	539.5	05370	: 27	C:300	1st Qu.:30	1st Qu.:200	male :339 [3,19) :279
Median :	1155.0	11000	: 27		Median :30	Median :200	NA's : 5 [19,Inf):162
Mean :	1192.7	05358	: 13		Mean :30	Mean :200	NA's : 1
3rd Qu.:	1808.0	05162	: 12		3rd Qu.:30	3rd Qu.:200	
Max.	:2542.8	05382	: 12		Max. :30	Max. :200	
		(Other):	511				

The associated event coordinates are residence postcode centroids, projected in the *European Terrestrial Reference System 1989* (in kilometer units) to enable Euclidean geometry. See the `spTransform`-methods in package **rgdal** (Bivand, Keitt, and Rowlingson 2017) for how to project latitude and longitude coordinates into a planar coordinate reference system (CRS). The data frame associated with these spatial coordinates (\mathbf{s}_i) contains a number of required variables and additional event marks (in the notation of Section 1: $\{(t_i, [\mathbf{s}_i], k_i, \tau_i, \delta_i, \mathbf{m}_i) : i = 1, \dots, n\}$). For the IMD data, the event `time` is measured in days since the beginning of the observation period 2002–2008 and is subject to a tie-breaking procedure (described later). The `tile` column refers to the region of the spatio-temporal grid where the event occurred and here contains the official key of the administrative district of the patient's residence. There are two `types` of events labeled as "B" and "C", which refer to the serogroups of

the two meningococcal finetypes *B:P1.7-2,4:F1-5* and *C:P1.5,2:F3-3* contained in the data. The `eps.t` and `eps.s` columns specify upper limits for temporal and spatial interaction, respectively. Here, the infectious period is assumed to last a maximum of 30 days and spatial interaction is limited to a 200 km radius for all cases. The latter has numerical advantages for a Gaussian interaction function f with a relatively small standard deviation. For a power-law kernel, however, this restriction will be dropped to enable occasional long-range transmission. The last two data attributes displayed in the above `event` summary are covariates from the case reports: the gender and age group of the patient.

For the observation region W , we use a polygon representation of Germany's boundary. Since the observation region defines the integration domain in the point process log-likelihood (4), the more detailed the polygons of W are the longer it will take to fit a `twinstim`. It is thus advisable to sacrifice some shape details for speed by reducing the polygon complexity, e.g., by applying one of the simplification methods available at MapShaper.org. Alternative tools in R are `spatstat`'s `simplify.owin` procedure (Baddeley, Turner, and Rubak 2017) and the function `thinnedSpatialPoly` in package `maptools` (Bivand and Lewin-Koh 2017), which implements the Douglas-Peucker reduction method. The `surveillance` package already contains a simplified representation of Germany's boundaries:

```
R> load(system.file("shapes", "districtsD.RData", package = "surveillance"))
```

This file contains both the `SpatialPolygonsDataFrame` `districtsD` of Germany's 413 administrative districts as at January 1, 2009, as well as their union `stateD`. These boundaries are projected in the same CRS as the `events` data.

The `stgrid` input for the endemic model component is a data frame with (time-varying) area-level covariates, e.g., socio-economic or ecological characteristics. In our example:

	start	stop	tile	area	popdensity
1	0	31	01001	56.4	1557.1
2	0	31	01002	118.7	1996.6
3	0	31	01003	214.2	987.6
...
34690	2526	2557	16075	1148.5	79.2
34691	2526	2557	16076	843.5	133.6
34692	2526	2557	16077	569.1	181.5

Numeric (`start,stop`] columns index the time periods and the factor variable `tile` identifies the regions of the grid. Note that the given time intervals (here: months) also define the resolution of possible time trends and seasonality of the piecewise constant endemic intensity. We choose monthly intervals to reduce package size and computational cost compared to the weekly resolution originally used by Meyer *et al.* (2012) and Meyer and Held (2014). The above `stgrid` data frame thus consists of 7 (years) times 12 (months) blocks of 413 (districts) rows each. The `area` column gives the area of the respective `tile` in square kilometers (compatible with the CRS used for `events` and W). A geographic representation of the regions in `stgrid` is not required for model estimation, and is thus not part of the `epidataCS` class. In our example, the area-level data only consists of the population density `popdensity`, whereas Meyer *et al.* (2012) additionally incorporated (lagged) weekly influenza counts by district as a time-dependent covariate.

2.2. Data handling and visualization

The generated `epidataCS` object `imdepi` is a simple list of the checked ingredients `events`, `stgrid`, `W`, `qmatrix`. Several methods for data handling and visualization are available for such objects as listed in Table 1 and briefly presented in the remainder of this section.

Display	Subset	Extract	Modify	Convert
<code>print</code>	<code>[</code>	<code>nobs</code>	<code>update</code>	<code>as.epidata</code>
<code>summary</code>	<code>head</code>	<code>marks</code>	<code>untie</code>	<code>epidataCS2sts</code>
<code>plot</code>	<code>tail</code>			
<code>animate</code>	<code>subset</code>			
<code>as.stepfun</code>				

Table 1: Generic and *non-generic* functions applicable to `epidataCS` objects.

Printing an `epidataCS` object presents some metadata and the first 6 events by default:

```
R> imdepi
```

```
Observation period: 0 - 2557
Observation window (bounding box): [4031, 4672] x [2684, 3550]
Spatio-temporal grid (not shown): 84 time blocks x 413 tiles
Types of events: "B" "C"
Overall number of events: 636
```

```
  coordinates    time  tile type eps.t eps.s  sex  agegrp BLOCK start popdensity
1 (4112, 3203)  0.2117 05554   B    30   200  male  [3,19)    1     0      260.9
2 (4123, 3077)  0.7124 05382   C    30   200  male  [3,19)    1     0      519.4
3 (4412, 2916)  5.5910 09574   B    30   200 female [19,Inf)    1     0      209.4
4 (4203, 2880)  7.1170 08212   B    30   200 female  [3,19)    1     0     1665.6
5 (4128, 3223) 22.0595 05554   C    30   200  male  [3,19)    1     0      260.9
6 (4090, 3178) 24.9544 05170   C    30   200  male  [3,19)    1     0      454.7
[....]
```

During conversion to `epidataCS`, the last three columns `BLOCK` (time interval index), `start` and `popdensity` have been merged from the checked `stgrid` to the `events` data frame. The event marks including time and location can be extracted in a standard data frame by `marks(imdepi)` – inspired by package **spatstat** – and this is summarized by `summary(imdepi)`. The number of potential sources of infection per event (denoted `|.sources|` in the above output) is additionally summarized. It is determined by the events' maximum ranges of interaction `eps.t` and `eps.s`. The event-specific set of potential sources is stored in the (hidden) list `imdepi$events$.sources` (events are referenced by row index), and the event-specific numbers of potential sources are stored in the summarized object as `simdepi$nSources`.

A simple plot of the number of infectives as a function of time (Figure 1) can be obtained by the step function converter:

```
R> plot(as.stepfun(imdepi), xlim = summary(imdepi)$timeRange, xaxs = "i",
+       xlab = "Time [days]", ylab = "Current number of infectives", main = "")
```

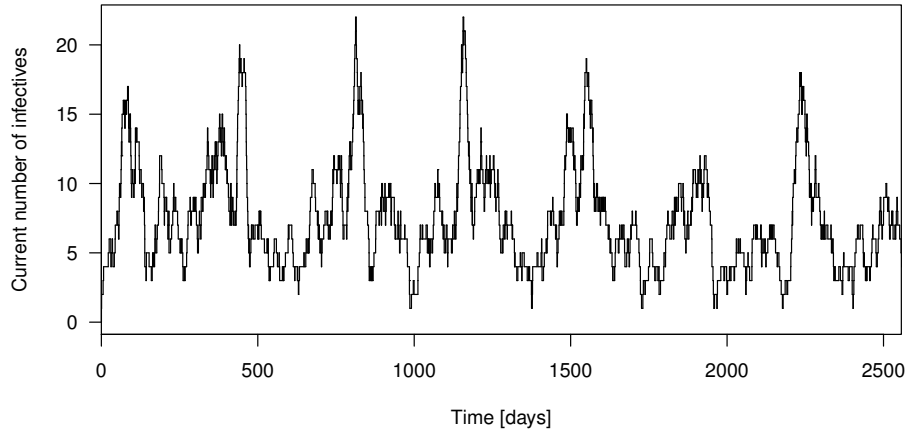


Figure 1: Time course of the number of infectives assuming infectious periods of 30 days.

The `plot`-method for `epidataCS` offers aggregation of the events over time or space:

```
R> plot(imdepi, "time", col = c("indianred", "darkblue"), ylim = c(0, 20))
R> plot(imdepi, "space", lwd = 2,
+       points.args = list(pch = c(1, 19), col = c("indianred", "darkblue")))
R> layout.scalebar(imdepi$W, scale = 100, labels = c("0", "100 km"), plot = TRUE)
```

The time-series plot (Figure 2a) shows the monthly aggregated number of cases by finetype in a stacked histogram as well as each type's cumulative number over time. The spatial plot (Figure 2b) shows the observation window W with the locations of all cases (by type), where the areas of the points are proportional to the number of cases at the respective location. Additional shading by the population is possible and exemplified in `help("plot.epidataCS")`.

The above static plots do not capture the space-time dynamics of epidemic spread. An animation may provide additional insight and can be produced by the corresponding `animate`-method. For instance, to look at the first year of the B-type in a weekly sequence of snapshots in a web browser (using facilities of the **animation** package of Xie 2017):

```
R> animation::saveHTML(
+   animate(subset(imdepi, type == "B"), interval = c(0, 365), time.spacing = 7),
+   nmax = Inf, interval = 0.2, loop = FALSE,
+   title = "Animation of the first year of type B events")
```

Selecting events from `epidataCS` as for the animation above is enabled by the `[-` and `subset`-methods, which return a new `epidataCS` object containing only the selected `events`.

A limited data sampling resolution may lead to tied event times or locations, which are in conflict with a continuous spatio-temporal point process model. For instance, a temporal residual analysis would suggest model deficiencies (Meyer *et al.* 2012, Figure 4), and a power-law kernel for spatial interaction may diverge if there are events with zero distance to potential source events (Meyer and Held 2014). The function `untie` breaks ties by random shifts. This has already been applied to the event *times* in the provided `imdepi` data by subtracting a $U(0,1)$ -distributed random number from the original dates. The event *coordinates* in the IMD data are subject to interval censoring at the level of Germany's postcode regions. A possible

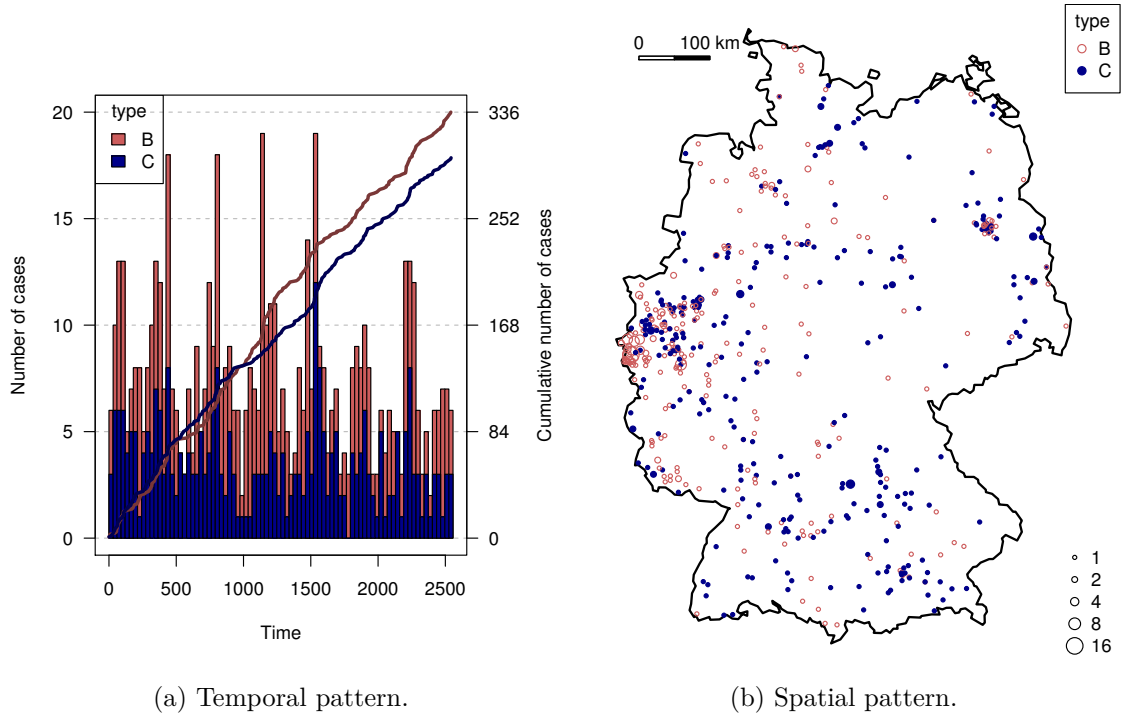


Figure 2: Occurrence of the two finetypes viewed in the temporal and spatial dimensions.

replacement for the given centroids would thus be a random location within the corresponding postcode area. Lacking a suitable shapefile, [Meyer and Held \(2014\)](#) shifted all locations by a random vector with length up to half the observed minimum spatial separation:

```
R> eventDists <- dist(coordinates(imdepi$events))
R> (minsep <- min(eventDists[eventDists > 0]))

[1] 1.173

R> set.seed(321)
R> imdepi_untied <- untie(imdepi, amount = list(s = minsep / 2))
```

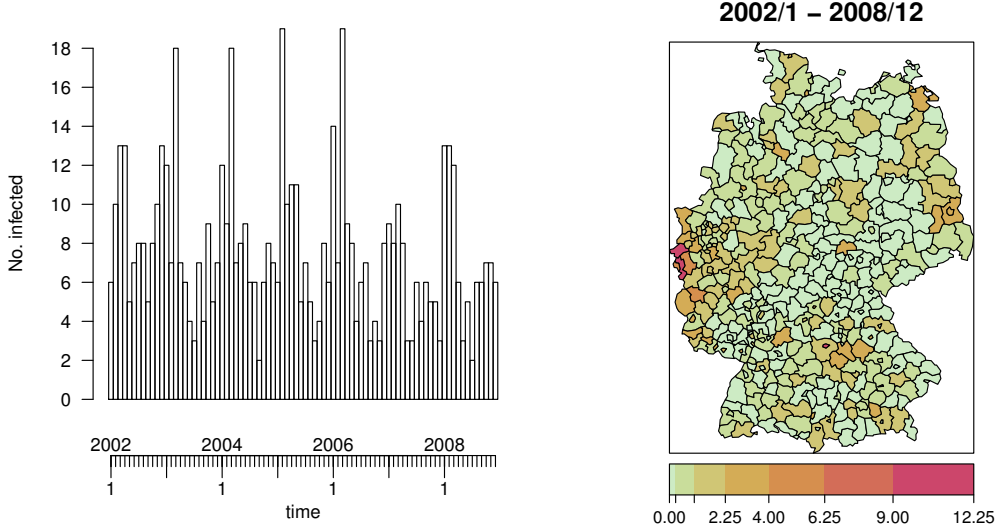
Note that random tie-breaking requires sensitivity analyses as discussed by [Meyer and Held \(2014\)](#), but skipped here for the sake of brevity.

The `update`-method is useful to change the values of the maximum interaction ranges `eps.t` and `eps.s`, since it takes care of the necessary updates of the hidden auxiliary variables in an `epidataCS` object. For unbounded spatial interaction:

```
R> imdepi_untied_infeps <- update(imdepi_untied, eps.s = Inf)
```

Last but not least, `epidataCS` can be converted to the other classes `epidata` (described in `vignette("twinSIR")`) and `sts` (described in, e.g., `vignette("hhh4_spacetime")`) by aggregation. The method `as.epidata.epidataCS` aggregates events by region (`tile`), and the function `epidataCS2sts` yields counts by region and time interval. The data could then, e.g., be analyzed by the areal time-series model presented in `vignette("hhh4_spacetime")`. We can also use visualization tools of the `sts` class, e.g., to produce Figure 3:

```
R> imdsts <- epidataCS2sts(imdepi, freq = 12, start = c(2002, 1), tiles = districtsD)
R> plot(imdsts, type = observed ~ time)
R> plot(imdsts, type = observed ~ unit, population = districtsD$POPULATION / 100000)
```



(a) Time series of monthly counts.

(b) Disease incidence (per 100 000 inhabitants).

Figure 3: IMD cases (joint types) aggregated as an `sts` object by month and district.

3. Modeling and inference

Having prepared the data as an object of class `epidataCS`, the function `twinstim` can be used to perform likelihood inference for conditional intensity models of the form (1). The main arguments for `twinstim` are the formulae of the **endemic** and **epidemic** linear predictors ($\nu_{[s][t]} = \exp(\text{endemic})$ and $\eta_j = \exp(\text{epidemic})$), and the spatial and temporal interaction functions `siaf` (f) and `tiaf` (g), respectively. Both formulae are parsed internally using the standard `model.frame` toolbox from package `stats` and thus can handle factor variables and interaction terms. While the **endemic** linear predictor incorporates covariates from `stgrid`, the **epidemic** formula may use both `stgrid` variables and event marks to be associated with the force of infection. For the interaction functions, several alternatives are predefined as listed in Table 2. They are applicable out-of-the-box and illustrated as part of the following modeling exercise for the IMD data. Own interaction functions can also be implemented following the structure described in `help("siaf")` and `help("tiaf")`, respectively.

3.1. Basic example

To illustrate statistical inference with `twinstim`, we will estimate several models for the simplified and “untied” IMD data presented in Section 2. In the endemic component, we include the district-specific population density as a multiplicative offset, a (centered) time trend, and a sinusoidal wave of frequency $2\pi/365$ to capture seasonality, where the `start` variable from `stgrid` measures time:

Spatial (siaf.*)	Temporal (tiaf.*)
constant	constant
gaussian	exponential
powerlaw	step
powerlawL	
step	
student	

Table 2: Predefined spatial and temporal interaction functions.

```
R> (endemic <- addSeason2formula(~offset(log(popdensity)) + I(start / 365 - 3.5),
+   period = 365, timevar = "start"))
```

```
~offset(log(popdensity)) + I(start/365 - 3.5) + sin(2 * pi *
  start/365) + cos(2 * pi * start/365)
```

See [Held and Paul \(2012, Section 2.2\)](#) for how such sine/cosine terms reflect seasonality. Because of the aforementioned integrations in the log-likelihood (4), it is advisable to first fit an endemic-only model to obtain reasonable start values for more complex epidemic models:

```
R> imdfit_endemic <- twinstim(endemic = endemic, epidemic = ~0,
+   data = imdepi_untied, subset = !is.na(agegrp))
```

We exclude the single case with unknown age group from this analysis since we will later estimate an effect of the age group on the force of infection.

Many of the standard functions to access model fits in R are also implemented for `twinstim` fits (see Table 3). For example, we can produce the usual model summary:

```
R> summary(imdfit_endemic)
```

Call:

```
twinstim(endemic = endemic, epidemic = ~0, data = imdepi_untied,
  subset = !is.na(agegrp))
```

Coefficients of the endemic component:

	Estimate	Std. Error	z value	Pr(> z)
h.(Intercept)	-20.3683	0.0419	-486.24	< 2e-16 ***
h.I(start/365 - 3.5)	-0.0444	0.0200	-2.22	0.027 *
h.sin(2 * pi * start/365)	0.2733	0.0576	4.75	2.0e-06 ***
h.cos(2 * pi * start/365)	0.3509	0.0581	6.04	1.5e-09 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

No epidemic component.

AIC: 19166

Log-likelihood: -9579

Because of the aforementioned equivalence of the endemic component with a Poisson regression model, the coefficients can be interpreted as log rate ratios in the usual way. For instance, the endemic rate is estimated to decrease by $1 - \exp(\text{coef}(\text{imdfit_endemic})[2]) = 4.3\%$ per year. Coefficient correlations can be retrieved by the argument `correlation = TRUE` in the `summary` call just like for `summary.glm`, but may also be extracted via the standard `cov2cor(vcov(imdfit_endemic))`.

Display	Extract	Modify	Other
<code>print</code>	<code>nobs</code>	<code>update</code>	<code>simulate</code>
<code>summary</code>	<code>vcov</code>	<code>add1</code>	<code>all.equal</code>
<code>xtable</code>	<code>coeflist</code>	<code>drop1</code>	<code>epitest</code>
<code>plot</code>	<code>logLik</code>	<code>stepComponent</code>	
<code>intensityplot</code>	<code>extractAIC</code>		
<code>iafplot</code>	<code>profile</code>		
<code>checkResidualProcess</code>	<code>residuals</code>		
	<code>terms</code>		
	<code>RO</code>		
	<code>intensity.twinstim</code>		
	<code>simpleRO</code>		

Table 3: Generic and *non-generic* functions applicable to `twinstim` objects. Note that there is no need for specific `coef`, `confint`, AIC or BIC methods, since the respective default methods from package `stats` apply outright.

We now update the endemic model to take additional spatio-temporal dependence between events into account. Infectivity shall depend on the meningococcal finetype and the age group of the patient, and is assumed to be constant over time (default), $g(t) = \mathbb{1}_{(0,30]}(t)$, with a Gaussian distance-decay $f(x) = \exp\{-x^2/(2\sigma^2)\}$. This model was originally selected by Meyer *et al.* (2012) and can be fitted as follows:

```
R> imdfit_Gaussian <- update(imdfit_endemic, epidemic = ~type + agegrp,
+   siaf = siaf.gaussian(), start = c("e.(Intercept)" = -12.5, "e.siaf.1" = 2.75),
+   control.siaf = list(F = list(adapt = 0.25), Deriv = list(nGQ = 13)),
+   cores = 2 * (.Platform$OS.type == "unix"))
```

To reduce the runtime of this example, we specified convenient `start` values for some parameters (others start at 0) and set `control.siaf` with a rather low number of nodes for the cubature of $f(\|s\|)$ in the log-likelihood (via the midpoint rule) and $\frac{\partial f(\|s\|)}{\partial \log \sigma}$ in the score function (via product Gauss cubature). On Unix-alikes, these numerical integrations can be performed in parallel using the “multicore” functions `mclapply` *et al.* from the base package `parallel`, here with `cores = 2` processes.

Table 4 shows the output of `twinstim`’s `xtable` method (Dahl 2016), which provides rate ratios for the endemic and epidemic effects. The alternative `toLatex` method simply translates the `summary` table of coefficients to L^AT_EX without `exp`-transformation. On the subject-matter level, we can conclude from Table 4 that the meningococcal finetype of serogroup C is less than half as infectious as the B-type, and that patients in the age group 3 to 18 years are estimated to cause twice as many secondary infections as infants aged 0 to 2 years.

	RR	95% CI	p-value
h.I(start/365 - 3.5)	0.955	0.91–1.00	0.039
h.sin(2 * pi * start/365)	1.243	1.09–1.41	0.0008
h.cos(2 * pi * start/365)	1.375	1.21–1.56	<0.0001
e.typeC	0.402	0.24–0.68	0.0007
e.agegrp[3,19)	2.000	1.06–3.78	0.033
e.agegrp[19,Inf)	0.776	0.32–1.91	0.58

Table 4: Estimated rate ratios (RR) and associated Wald confidence intervals (CI) for endemic (h.) and epidemic (e.) terms. This table was generated by `xtable(imdfit_Gaussian)`.

3.2. Model-based effective reproduction numbers

The event-specific reproduction numbers (2) can be extracted from fitted `twinstim` objects via the `R0` method. For the above IMD model, we obtain the following mean numbers of secondary infections by finetype:

```
R> R0_events <- R0(imdfit_Gaussian)
R> tapply(R0_events, marks(imdepi_untied)[names(R0_events), "type"], mean)

      B      C
0.21614 0.09576
```

Confidence intervals can be obtained via Monte Carlo simulation, where Equation 2 is repeatedly evaluated with parameters sampled from the asymptotic multivariate normal distribution of the maximum likelihood estimate. For this purpose, the `R0`-method takes an argument `newcoef`, which is exemplified in `help("R0")`.

3.3. Interaction functions

Figure 4 shows several estimated spatial interaction functions, which can be plotted by, e.g., `plot(imdfit_Gaussian, which = "siaf")`.

The estimated standard deviation $\hat{\sigma}$ of the Gaussian kernel is:

```
R> exp(cbind("Estimate" = coef(imdfit_Gaussian)["e.siaf.1"],
+           confint(imdfit_Gaussian, parm = "e.siaf.1")))

      Estimate 2.5 % 97.5 %
e.siaf.1      16 13.65 18.75
```

Meyer and Held (2014) found that a power-law decay of spatial interaction is more appropriate than a Gaussian kernel to describe the spread of human infectious diseases. The power-law kernel concentrates on short-range interaction, but also exhibits a heavier tail reflecting occasional transmission over large distances. To use the power-law kernel $f(x) = (x + \sigma)^{-d}$, we switch to the prepared `epidataCS` object with `eps.s = Inf` and update the previous Gaussian model as follows:

```
R> imdfit_powerlaw <- update(imdfit_Gaussian, data = imdepi_untied_infeps,
+   siaf = siaf.powerlaw(), control.siaf = NULL,
+   start = c("e.(Intercept)" = -6.2, "e.siaf.1" = 1.5, "e.siaf.2" = 0.9))
```

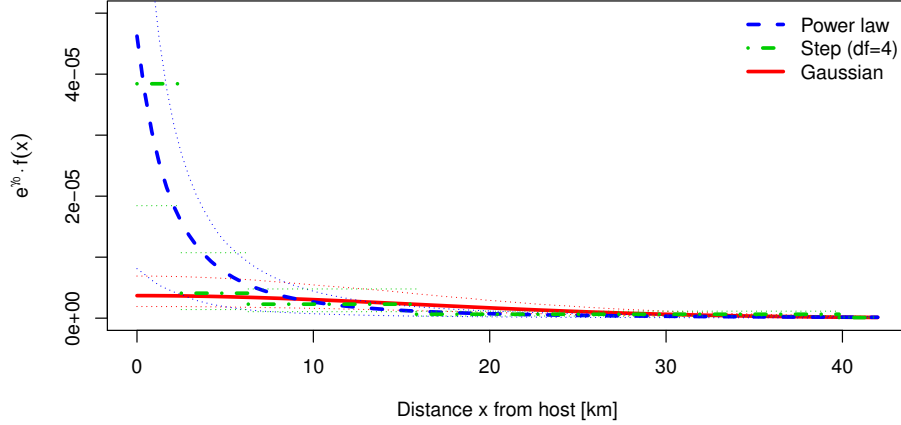


Figure 4: Various estimates of spatial interaction (scaled by the epidemic intercept γ_0).

The estimated power-law parameters ($\hat{\sigma}, \hat{d}$) are:

```
R> exp(cbind("Estimate" = coef(imdfit_powerlaw)[c("e.siaf.1", "e.siaf.2")],
+          confint(imdfit_powerlaw, parm = c("e.siaf.1", "e.siaf.2"))))
```

	Estimate	2.5 %	97.5 %
e.siaf.1	4.640	1.818	11.841
e.siaf.2	2.487	1.808	3.423

Table 2 also lists the step function kernel as an alternative, which is particularly useful for two reasons. First, it is a more flexible approach since it estimates interaction between the given knots without assuming an overall functional form. Second, the spatial integrals in the log-likelihood can be computed analytically for the step function kernel, which therefore offers a quick estimate of spatial interaction. We update the Gaussian model to use four steps at log-equidistant knots up to an interaction range of 100 km:

```
R> imdfit_step4 <- update(imdfit_Gaussian, data = imdepi_untied_infeps,
+   siaf = siaf.step(exp(1:4 * log(100) / 5), maxRange = 100), control.siaf = NULL,
+   start = c("e.(Intercept)" = -10, setNames(-2:-5, paste0("e.siaf.", 1:4))))
```

Figure 4 suggests that the estimated step function is in line with the power law.

For the temporal interaction function $g(t)$, model updates and plots are similarly possible, e.g., using `update(imdfit_Gaussian, tiaf = tiaf.exponential())`. However, the events in the IMD data are too rare to infer the time-course of infectivity with confidence.

3.4. Model selection

```
R> AIC(imdfit_endemic, imdfit_Gaussian, imdfit_powerlaw, imdfit_step4)
```

	df	AIC
imdfit_endemic	4	19166
imdfit_Gaussian	9	18967
imdfit_powerlaw	10	18940
imdfit_step4	12	18933

Akaike’s Information Criterion (AIC) suggests superiority of the power-law vs. the Gaussian model and the endemic-only model. The more flexible step function yields the best AIC value but its shape strongly depends on the chosen knots and is not guaranteed to be monotonically decreasing. The function `stepComponent` – a wrapper around the `step` function from `stats` – can be used to perform AIC-based stepwise selection within a given model component.

3.5. Model diagnostics

The element `"fittedComponents"` of a `twinstim` object contains the endemic and epidemic values of the estimated intensity at each event occurrence. However, plots of the conditional intensity (and its components) as a function of location or time provide more insight into the fitted process. Evaluation of `intensity.twinstim` requires the model environment to be stored with the fit. By default, `model = FALSE` in `twinstim`, but if the data are still available, the model environment can also be added afterwards using the convenient `update` method:

```
R> imdfit_powerlaw <- update(imdfit_powerlaw, model = TRUE)
```

Figure 5 shows an `intensityplot` of the fitted “ground” intensity $\sum_{k=1}^2 \int_W \hat{\lambda}(s, t, k) ds$:

```
R> intensityplot(imdfit_powerlaw, which = "total", aggregate = "time", types = 1:2)
```

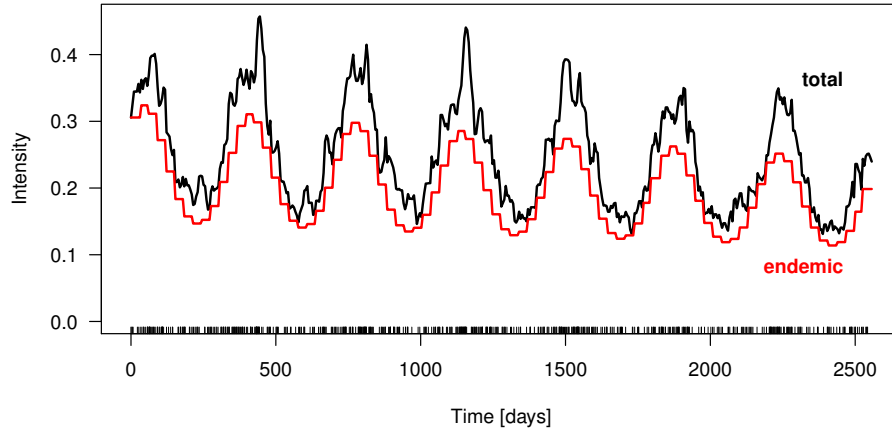


Figure 5: Fitted “ground” intensity process aggregated over space and both types.

The estimated endemic intensity component has also been added to the plot. It exhibits strong seasonality and a slow negative trend. The proportion of the endemic intensity is rather constant along time since no major outbreaks occurred. This proportion can be visualized separately by specifying `which = "endemic proportion"` in the above call.

Spatial `intensityplots` as in Figure 6 can be produced via `aggregate = "space"` and require a geographic representation of `stgrid`. The epidemic proportion is naturally high around clusters of cases and even more so if the population density is low.

Another diagnostic tool is the function `checkResidualProcess` (Figure 7), which transforms the temporal “residual process” in such a way that it exhibits a uniform distribution and lacks serial correlation if the fitted model describes the true CIF well (see [Ogata 1988](#), Section 3.3).

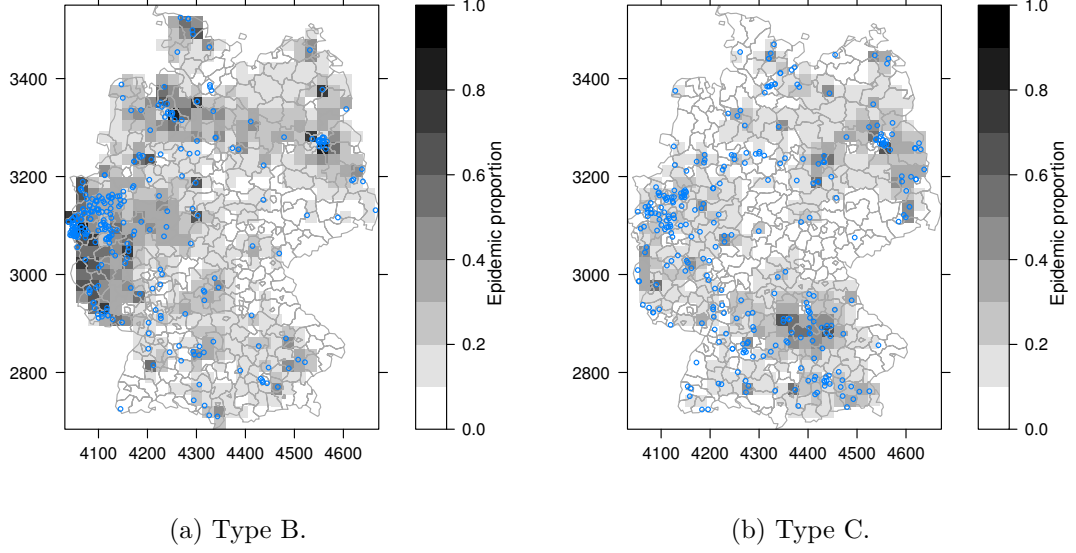


Figure 6: Epidemic proportion of the fitted intensity process accumulated over time by type.

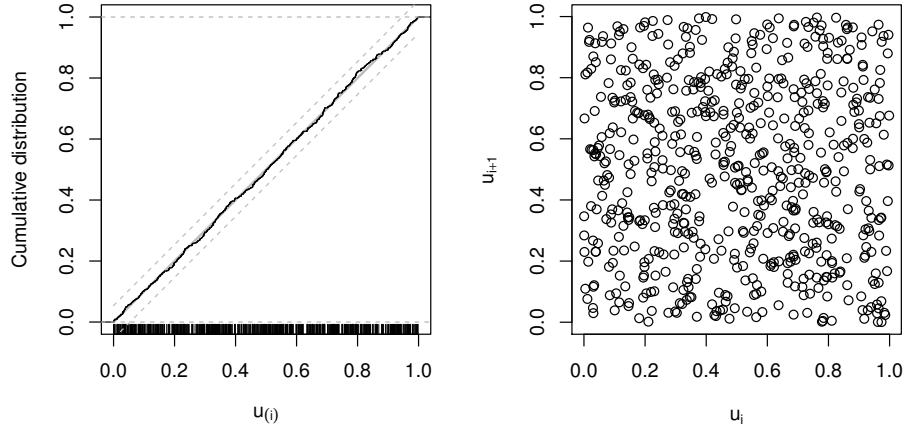


Figure 7: `checkResidualProcess(imdfit_powerlaw)`. The left plot shows the `ecdf` of the transformed residuals with a 95% confidence band obtained by inverting the corresponding Kolmogorov-Smirnov test (no evidence for deviation from uniformity). The right-hand plot suggests absence of serial correlation.

4. Simulation

To identify regions with unexpected IMD dynamics, Meyer *et al.* (2012) compared the observed numbers of cases by district to the respective 2.5% and 97.5% quantiles of 100 simulations from the selected model. Furthermore, simulations allow us to investigate the stochastic volatility of the endemic-epidemic process, to obtain probabilistic forecasts, and to perform parametric bootstrap of the spatio-temporal point pattern.

The simulation algorithm we apply is described in Meyer *et al.* (2012, Section 4). It requires a geographic representation of the `stgrid`, as well as functionality for sampling locations from the spatial kernel $f_2(\mathbf{s}) := f(\|\mathbf{s}\|)$. This is implemented for all predefined spatial interaction functions listed in Table 2. Event marks are by default sampled from their respective empirical distribution in the original data. The following code runs a single simulation over the last year based on the estimated power-law model:

```
R> imdsim <- simulate(imdfit_powerlaw, nsim = 1, seed = 1, t0 = 2191, T = 2555,
+   data = imdepi_untied_infeps, tiles = districtsD)
```

This yields an object of the class `simEpidataCS`, which extends `epidataCS`. It carries additional components from the generating model to enable an `R0`-method and `intensityplots` for simulated data. Figure 8 shows the cumulative number of cases from the simulation appended to the first six years of data.

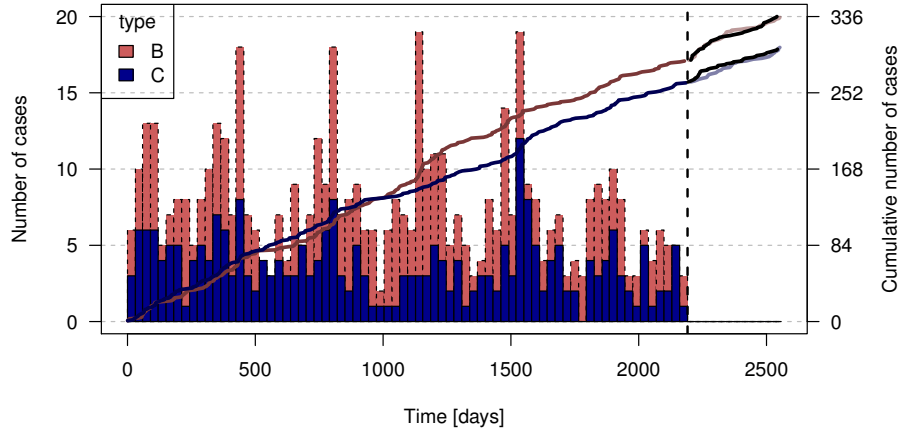


Figure 8: Simulation-based forecast of the cumulative number of cases by finetype in the last two years. The black lines correspond to the observed numbers.

A special feature of such simulated epidemics is that the source of each event is known:

```
R> table(imdsim$events$source > 0, exclude = NULL)
```

```
FALSE TRUE <NA>
   70   17    3
```

The stored `source` value is 0 for endemic events, `NA` for events of the prehistory but still infective at `t0`, and otherwise corresponds to the row index of the infective source.

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