

# A tutorial for Discriminant Analysis of Principal Components (DAPC) using *adeigenet* 1.3-4

Thibaut Jombart

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## Abstract

This vignette provides a tutorial for applying the Discriminant Analysis of Principal Components (DAPC [1]) using the *adeigenet* package [2] for the R software [3]. This methods aims to identify and describe genetic clusters, although it can in fact be applied to any quantitative data. We illustrate how to use `find.clusters` to identify clusters, and `dapc` to describe the relationships between these clusters. More advanced topics are then introduced, such as advanced graphics, assessing the stability of DAPC results and using supplementary individuals.

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# 1 Introduction

Investigating genetic diversity using multivariate approaches relies on finding synthetic variables built as linear combinations of alleles (i.e. new-variable =  $a_1\text{allele}_1 + a_2\text{allele}_2 + \dots$  where  $a_1, a_2$  etc. are real coefficients) and which reflect as well as possible the genetic variation amongst the studied individuals. However, most of the time we are not only interested in the diversity amongst individuals, but also and possibly more so in the diversity between groups of individuals. Typically, one will be analysing individual data to identify populations, or more largely genetic clusters, and then describe these clusters.

A problem occurring in traditional methods is they usually focus on the entire genetic variation. Genetic variability can be decomposed using a standard multivariate ANOVA model as:

$$\text{total variance} = (\text{variance between groups}) + (\text{variance within groups})$$

or more simply, denoting  $\mathbf{X}$  the data matrix:

$$VAR(\mathbf{X}) = B(\mathbf{X}) + W(\mathbf{X})$$

Usual approaches such as Principal Component Analysis (PCA) or Principal Coordinates Analysis (PCoA / MDS) focus on  $VAR(\mathbf{X})$ . That is, they only describe the global diversity, possibly overlooking differences between groups. On the contrary, DAPC optimizes  $B(\mathbf{X})$  while minimizing  $W(\mathbf{X})$ : it seeks synthetic variables, the *discriminant functions*, which show differences between groups as best as possible while minimizing variation within clusters.

## 2 Identifying clusters using `find.clusters`

### 2.1 Rationale

DAPC in itself requires prior groups to be defined. However, groups are often unknown or uncertain, and there is a need for identifying genetic clusters before describing them. This can be achieved using  $k$ -means, a clustering algorithm which finds a given number (say,  $k$ ) of groups maximizing the variation between groups,  $B(\mathbf{X})$ . To identify the optimal number of clusters,  $k$ -means is run sequentially with increasing values of  $k$ , and different clustering solutions are compared using Bayesian Information Criterion (BIC). Ideally, the optimal clustering solution should correspond to the lowest BIC. In practice, the 'best' BIC is often indicated by an elbow in the curve of BIC values as a function of  $k$ .

While  $k$ -means could be performed on the raw data, we prefer running the algorithm after transforming the data using PCA. This transformation has the major advantage of reducing the number of variables so as to speed up the clustering algorithm. Note that this does not imply a necessary loss of information since all the principal components (PCs) can be retained, and

therefore all the variation in the original data. In practice however, a reduced number of PCs is often sufficient to identify the existing clusters, while making the analysis essentially instantaneous.

## 2.2 In practice

Identification of the clusters is achieved by `find.clusters`. This function first transforms the data using PCA, asking the user to specify the number of retained PCs interactively unless the argument `n.pca` is provided. Then, it runs  $k$ -means algorithm (function `kmeans` from the *stats* package) with increasing values of  $k$ , unless the argument `n.clust` is provided, and computes associated summary statistics (by default, BIC). See `?find.clusters` for other arguments.

`find.clusters` is a generic function with methods for `data.frame`, objects with the class `genind` (usual genetic markers) and `genlight` (genome-wide SNP data). Here, we illustrate its use using a toy dataset simulated in [1], `dapcIllus`:

```
> library(adegenet)
> data(dapcIllus)
> class(dapcIllus)
```

```
[1] "list"
```

```
> names(dapcIllus)
```

```
[1] "a" "b" "c" "d"
```

`dapcIllus` is a list containing four datasets; we shall only use the first one:

```
> x <- dapcIllus$a
> x
```

```
#####
### Genind object ###
#####
- genotypes of individuals -

S4 class: genind
@call: read.fstat(file = file, missing = missing, quiet = quiet)

@tab: 600 x 140 matrix of genotypes

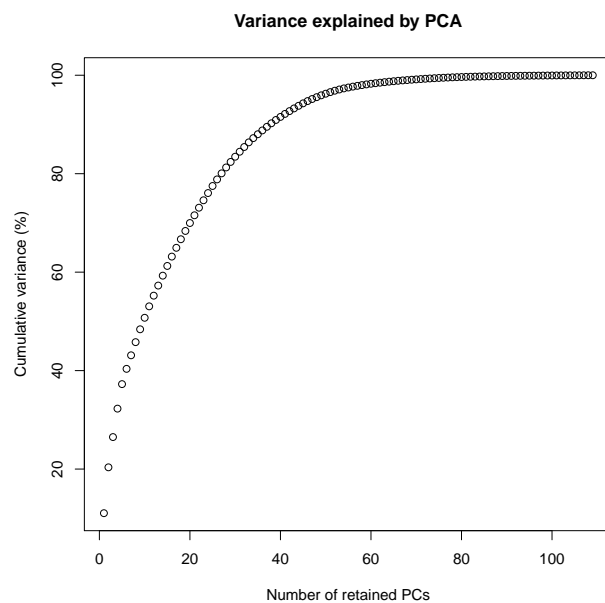
@ind.names: vector of 600 individual names
@loc.names: vector of 30 locus names
@loc.nall: number of alleles per locus
@loc.fac: locus factor for the 140 columns of @tab
@all.names: list of 30 components yielding allele names for each locus
@ploidy: 2
@type: codom

Optionnal contents:
@pop: factor giving the population of each individual
@pop.names: factor giving the population of each individual

@other: - empty -
```

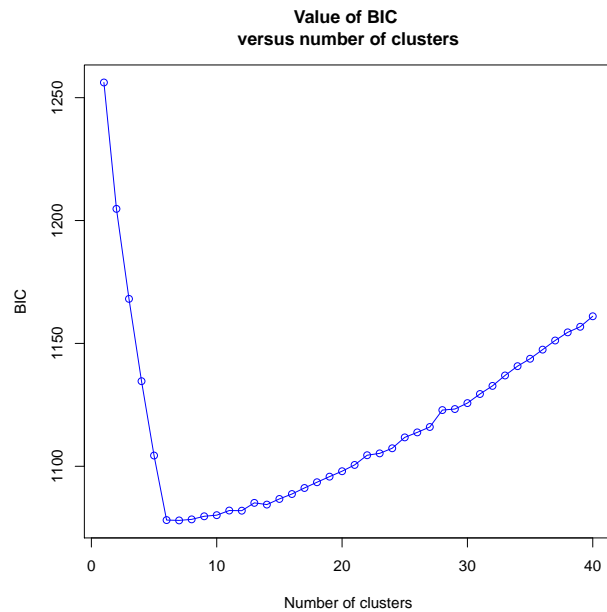
`x` is a dataset of 600 individuals simulated under an island model (6 islands) for 30 microsatellite markers. We use `find.clusters` to identify clusters, although true clusters are, in this case, known (and accessible using `pop(x)`). We specify that we want to evaluate up to  $k = 40$  groups (`max.n.clust=40`):

```
> grp <- find.clusters(x, max.n.clust=40)
```



The function displays a graph of cumulated variance explained by the eigenvalues of the PCA. Apart from computational time, there is no reason for keeping a small number of components; here, we keep all the information, specifying to retain 200 PCs (there are actually less PCs —around 110—, so all of them are kept).

Then, the function displays a graph of BIC values for increasing values of  $k$ :



This graph shows a clear decrease of BIC until  $k = 6$  clusters, after which BIC increases. In this case, the elbow in the curve also matches the smallest BIC, and clearly indicates 6 clusters should be retained. In practice, the choice is often trickier to make for empirical dataset.

The output of `find.clusters` is a list:

```
> names(grp)

[1] "Kstat" "stat"  "grp"   "size"

> head(grp$Kstat, 8)

NULL

> grp$stat

NULL

> head(grp$grp, 10)

001 002 003 004 005 006 007 008 009 010
  1   1   1   4   1   1   1   1   1   1
Levels: 1 2 3 4 5 6
```

```
> grp$size
```

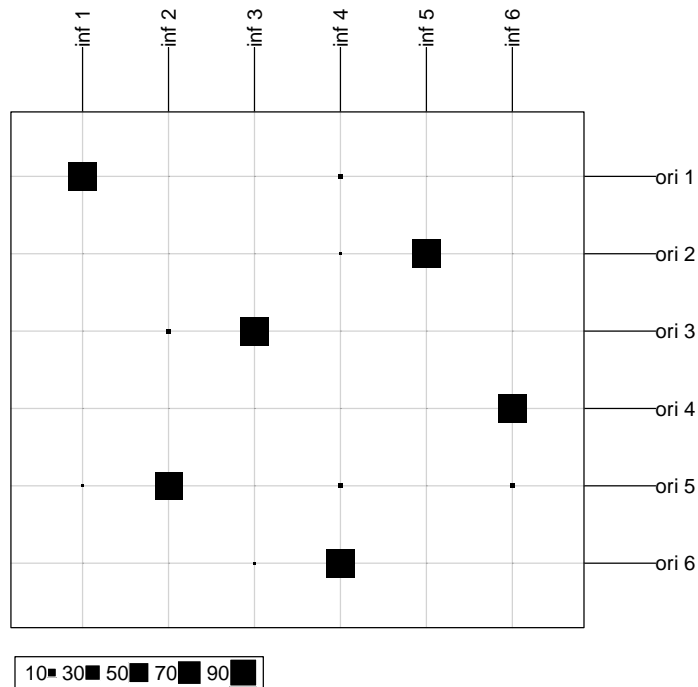
```
[1] 98 97 99 105 99 102
```

The components are respectively the chosen summary statistics (here, BIC) for different values of  $k$  (slot `Kstat`), the selected number of clusters and the associated BIC (slot `stat`), the group memberships (slot `grp`) and the group sizes (slot `size`). Here, since we know the actual groups, we can check how well they have been retrieved by the procedure. Actual groups are accessed using `pop`:

```
> table(pop(x), grp$grp)
```

	1	2	3	4	5	6
1	97	0	0	3	0	0
2	0	0	0	1	99	0
3	0	2	98	0	0	0
4	0	0	0	0	0	100
5	1	95	0	2	0	2
6	0	0	1	99	0	0

```
> table.value(table(pop(x), grp$grp), col.lab=paste("inf", 1:6), row.lab=paste("ori", 1:6))
```

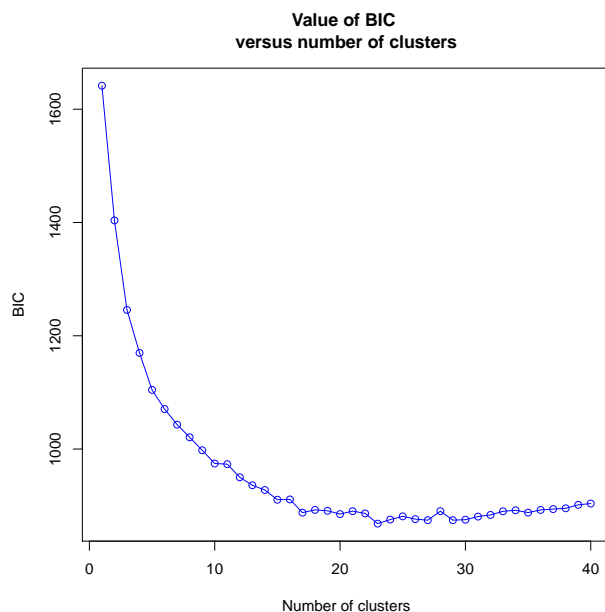


Rows correspond to actual groups ("ori"), while columns correspond to inferred groups ("inf"). Here, we can see that original groups have nearly been perfectly identified by the method.

## 2.3 How many clusters are there really in the data?

Although the most frequently asked when trying to find clusters in genetic data, this question is equally often meaningless. Clustering algorithms help making a caricature of a complex reality, which is most of the time far from following known population genetics models. Therefore, we are rarely looking for actual panmictic populations from which the individuals have been drawn. Genetic clusters can be biologically meaningful structures and reflect interesting biological processes, but they are still models.

A slightly different but probably more meaningful question would be: "How many clusters are useful to describe the data?". A fundamental point in this question is that clusters are merely tools used to summarise and understand the data. There is no longer a "true  $k$ ", but some values of  $k$  are better, more efficient summaries of the data than others. For instance, in the following case:



, the concept of "true  $k$ " is fairly hypothetical. This does not mean that clustering algorithms should necessarily be discarded, but surely the reality is more complex than a few clear-cut, isolated populations. What the BIC decrease says is that 10-20 clusters would provide useful summaries of the data. The actual number retained is merely a question of personal taste.



## 3 Describing clusters using dapc

### 3.1 Rationale

DAPC aims to provide an efficient description of genetic clusters using a few synthetic variables. These are constructed as linear combinations of the original variables (alleles) which have the largest between-group variance and the smallest within-group variance. Coefficients of the alleles used in the linear combination are called *loadings*, while the synthetic variables are themselves referred to as *discriminant functions*.

Moreover, being based on the Discriminant Analysis, DAPC also provides membership probabilities of each individual for the different groups based on the retained discriminant functions. While these are different from the admixture coefficients of software like STRUCTURE, they can still be interpreted as proximities of individuals to the different clusters. Membership probabilities also provide indications of how clear-cut genetic clusters are. Loose clusters will result in fairly flat distributions of membership probabilities of individuals across clusters, pointing to possible admixture.

Lastly, using the allele loadings, it is possible to represent new individuals (which have not participated to the analysis) onto the factorial planes, and derive membership probabilities as well. Such individuals are referred to as *supplementary individuals*.

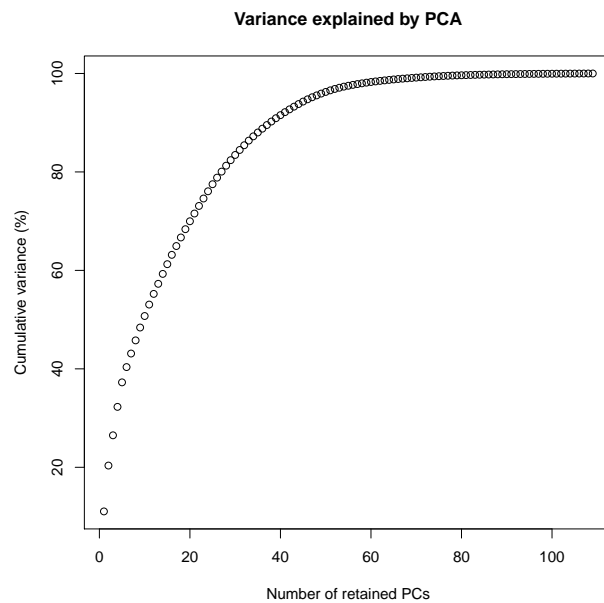
### 3.2 In practice

DAPC is implemented by the function `dapc`, which first transforms the data using PCA, and then performs a Discriminant Analysis on the retained principal components. Like `find.clusters`, `dapc` is a generic function with methods for `data.frame`, and objects with the class `genind` (usual genetic markers) and `genlight` (genome wide SNP data).

We run the analysis on the previous toy dataset, using the inferred groups stored in `grp$grp`:

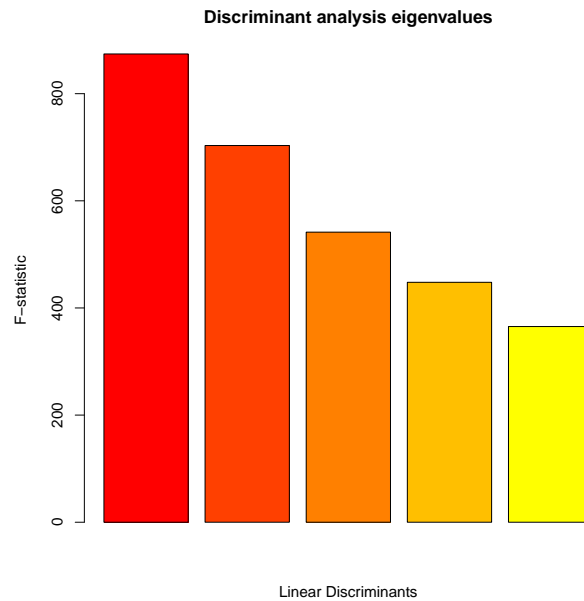
```
> dapc1 <- dapc(x, grp$grp)
```

The method displays the same graph of cumulated variance as in `find.cluster`. However, unlike *k*-means, DAPC can benefit from not using too many PCs. Indeed, retaining too many components with respect to the number of individuals can lead to over-fitting and unstability in the membership probabilities returned by the method (see section below about the stability of membership probabilities).



The bottomline is therefore retaining a few PCs without sacrificing too much information. Here, we can see that little information is gained by adding PCs after the first 40. We therefore retain 40 PCs.

Then, the method displays a barplot of eigenvalues for the discriminant analysis, asking for a number of discriminant functions to retain (unless argument `n.da` is provided).



For small number of clusters, all eigenvalues can be retained since all discriminant functions can be examined without difficulty. Whenever more (say, tens of) clusters are analysed, it is likely that the first few dimensions will carry more information than the others, and only those can then be retained and interpreted.

The object `dapc1` contains a lot of information:

```
> dapc1

#####
# Discriminant Analysis of Principal Components #
#####
class: dapc
$call: dapc.genind(x = x, pop = grp$grp, n.pca = 40, n.da = 100)

$n.pca: 40 first PCs of PCA used
$n.da: 5 discriminant functions saved
$var (proportion of conserved variance): 0.915

$eig (eigenvalues): 874.1 703.2 541.5 447.9 365.3  vector    length content
1 $eig      5      eigenvalues
2 $grp     600     prior group assignment
3 $prior    6      prior group probabilities
4 $assign  600     posterior group assignment
5 $pca.cent 140     centring vector of PCA
6 $pca.norm 140     scaling vector of PCA
7 $pca.eig 109     eigenvalues of PCA

data.frame  nrow ncol content
1 $tab      600  40   retained PCs of PCA
2 $means     6   40   group means
3 $loadings  40   5    loadings of variables
4 $ind.coord 600   5    coordinates of individuals (principal components)
```

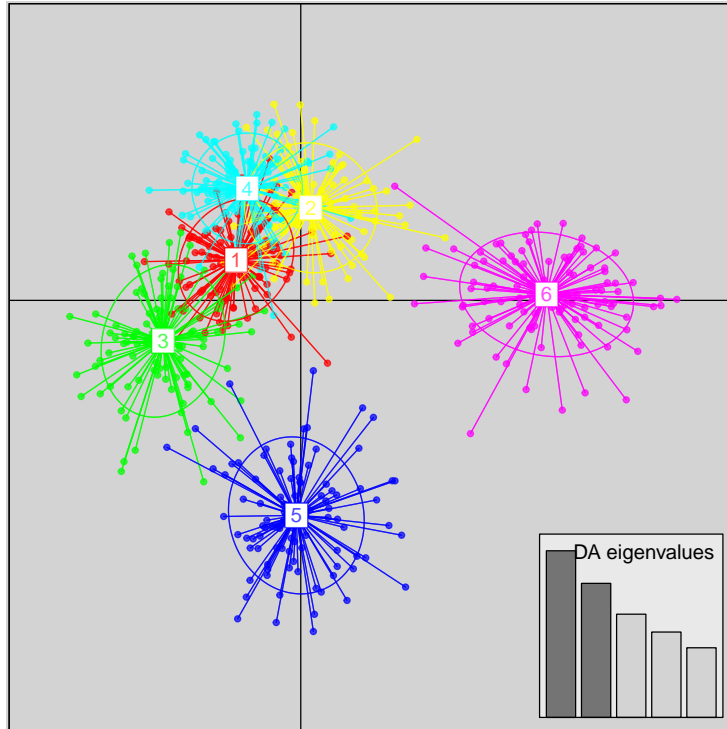
```

5 $grp.coord      6 5 coordinates of groups
6 $posterior      600 6 posterior membership probabilities
7 $pca.loadings  140 40 PCA loadings of original variables
8 $var.contr      140 5 contribution of original variables

```

For details about this content, please read the documentation (`?dapc`). Essentially, the slots `ind.coord` and `grp.coord` contain the coordinates of the individuals and of the groups used in scatterplots. Contributions of the alleles to each discriminant function are stored in the slot `var.contr`. Eigenvalues, corresponding to the ratio of the variance between groups over the variance within groups for each discriminant function, are stored in `eig`. Basic scatterplots can be obtained using the function `scatterplot`:

```
> scatter(dapc1)
```



The obtained graph represents the individuals as dots and the groups as inertia ellipses. Eigenvalues of the analysis are displayed in inset. These graphs are fairly easy to customize, as shown below.

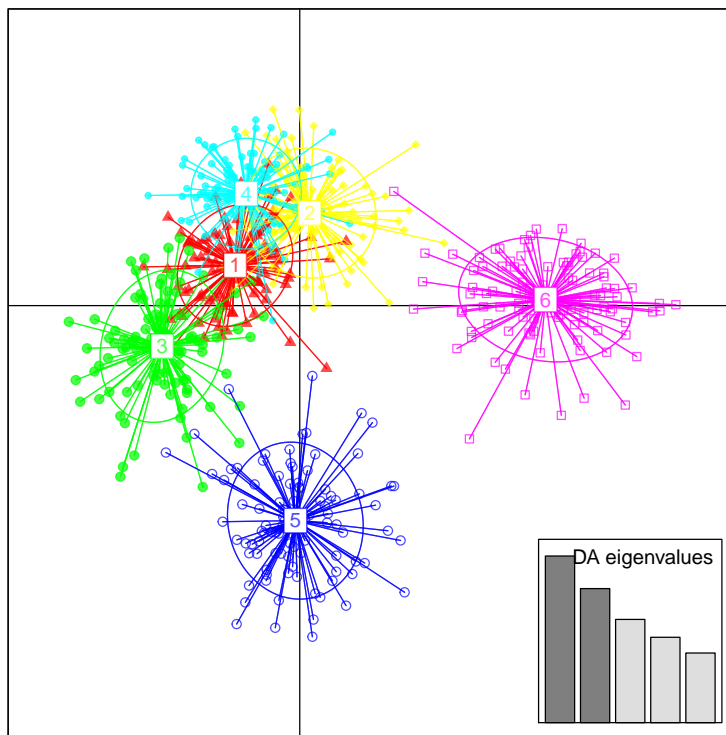
### 3.3 Customizing DAPC scatterplots

DAPC scatterplots are the main result of DAPC. It is therefore essential to ensure that information is displayed efficiently, and if possible to produce pretty figures. Possibilities are almost unlimited, and here we just illustrate a few possibilities offered by `scatter`. Note that `scatter` is a generic function,

with a dedicated method for objects produced by `dapc`. Documentation of this function can be accessed by typing `?scatter.dapc`.

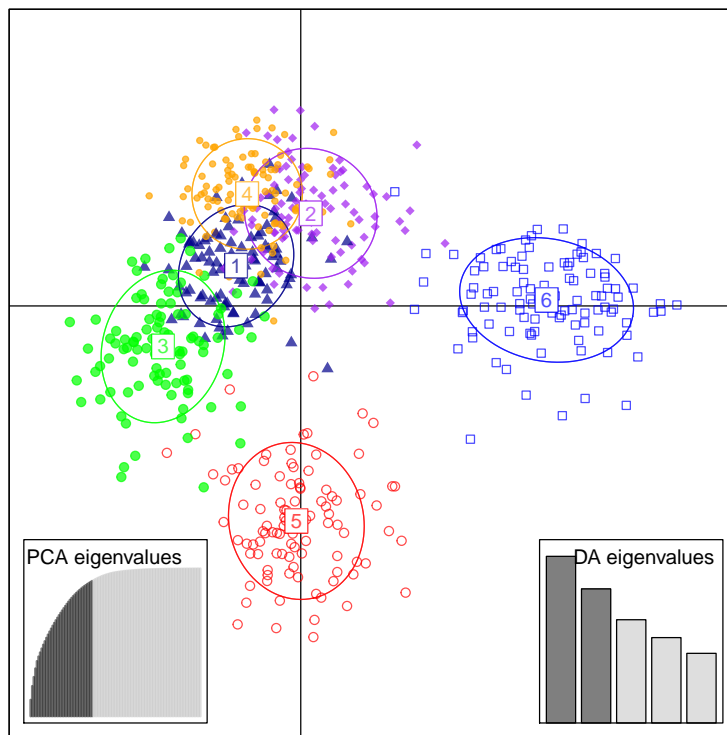
We illustrate some graphical possibilities trying to improve the display of the analysis presented in the previous section. While the default background (grey) allows to visualize rainbow colors (the default palette for the groups) more easily, it is not so pretty and is probably better removed for publication purpose. We also move the inset to a more appropriate place where it does not cover individuals, and use different symbols for the groups.

```
> scatter(dapc1, posi.da="bottomright", bg="white", pch=17:22)
```



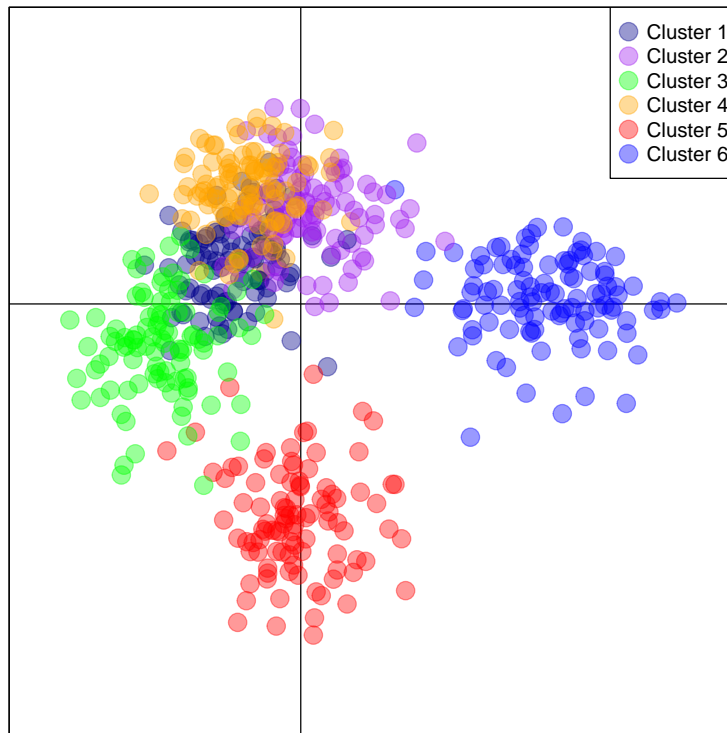
This is still not entirely satisfying: we need to define other colors more visible over a white background, and we can remove the segments linking the points to their ellipses:

```
> myCol <- c("darkblue","purple","green","orange","red","blue")
> scatter(dapc1, posi.da="bottomright", bg="white", pch=17:22, cstar=0, col=myCol, scree.pca=TRUE, posi.pca="bot
```



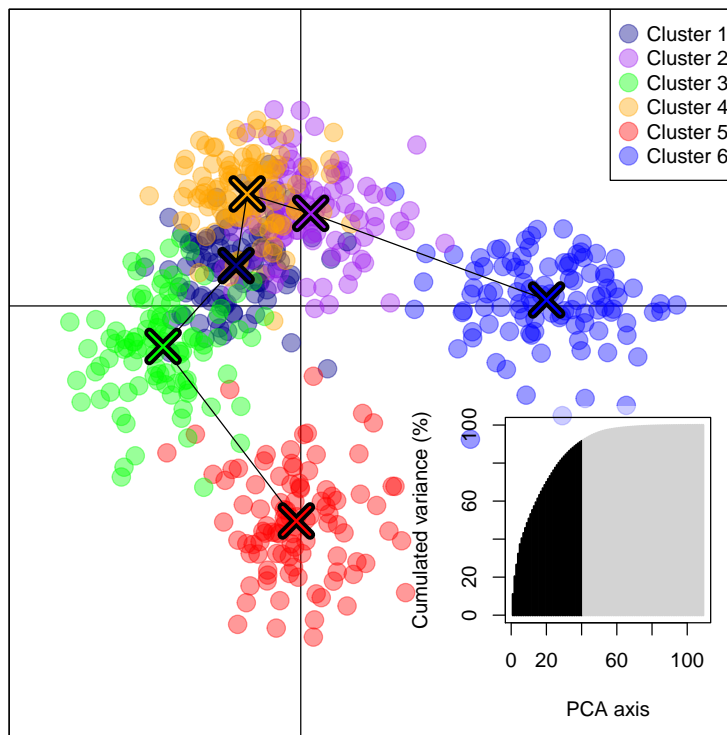
Another possibility is remove the labels within the ellipses and add a legend to the plot. We also use the same symbol for all individuals, but use bigger dots and transparent colours to have a better feel for the density of individuals on the factorial plane.

```
> scatter(dapc1, scree.da=FALSE, bg="white", pch=20, cell=0, cstar=0, col=myCol, solid=.4,
+         cex=3, clab=0, leg=TRUE, txt.leg=paste("Cluster",1:6))
```



We can also add a minimum spanning tree based on the (squared) distances between populations within the entire space. This allows one to bear in mind the actual proximities between populations inside the entire space, which are not always well represented in subsets of discriminant functions of lesser rank. We also indicate the centre of each group with crosses. Lastly, we remove the DAPC eigenvalues, not very useful in this case, and replace them manually by a graph of PCA eigenvalues retained in dimension-reduction step (retained eigenvalues in black, similar to using `scree.pca=TRUE`).

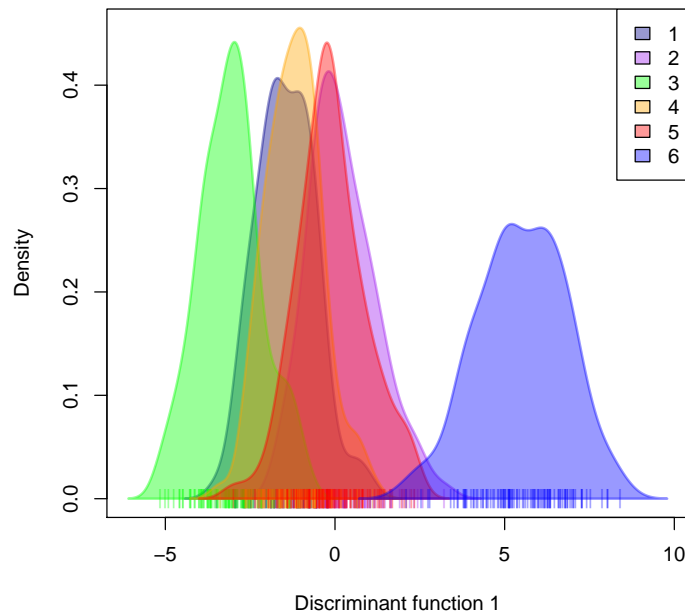
```
> scatter(dapc1, ratio.pca=0.3, bg="white", pch=20, cell=0, cstar=0, col=myCol, solid=.4,
+         cex=3, clab=0, mstree=TRUE, scree.da=FALSE,
+         posi.pca="bottomright", leg=TRUE, txt.leg=paste("Cluster",1:6))
> par(xpd=TRUE)
> points(dapc1$grp.coord[,1], dapc1$grp.coord[,2], pch=4, cex=3, lwd=8, col="black")
> points(dapc1$grp.coord[,1], dapc1$grp.coord[,2], pch=4, cex=3, lwd=2, col=myCol)
> myInset <- function(){
+   temp <- dapc1$pca.eig
+   temp <- 100* cumsum(temp)/sum(temp)
+   plot(temp, col=rep(c("black","lightgrey"), c(dapc1$n.pca,1000)), ylim=c(0,100),
+         xlab="PCA axis", ylab="Cumulated variance (%)", cex=1, pch=20, type="h", lwd=2)
+ }
> add.scatter(myInset(), posi="bottomright", inset=c(-0.03,-0.01), ratio=.28, bg=transp("white"))
```



Lastly, note that `scatter` can also represent a single discriminant function, which is especially useful when only one of these has been retained (e.g. in the case  $k = 2$ ). This is achieved by plotting the densities of individuals on a given discriminant function with different colors for different groups:

```
> scatter(dapc1,1,1, col=myCol, bg="white", scree.da=FALSE, legend=TRUE, solid=.4)
```





### 3.4 Interpreting variable contributions

In DAPC, the variable actually analyzed are principal components of a PCA. Loadings of these variables are generally uninformative, since PCs themselves do not all have straightforward interpretations. However, we can also compute contributions of the alleles, which can turn out to be very informative. In general, there are many alleles and their contribution is best plotted for a single discriminant function at a time.

Variable contributions are stored in the `var.contr` slot of a `dapc` object. They can be plotted using `loadingplot`. We illustrate this using the seasonal influenza dataset `H3N2`, which contains 1903 isolates genotyped for 125 SNPs located in the hemagglutinin segment (see `?H3N2`):

```
> data(H3N2)
> H3N2

#####
### Genind object ###
#####
- genotypes of individuals -

S4 class:  genind
@call:  .local(x = x, i = i, j = j, drop = drop)

@tab:  1903 x 334 matrix of genotypes

@ind.names:  vector of 1903 individual names
```

```

@loc.names: vector of 125 locus names
@loc.nall: number of alleles per locus
@loc.fac: locus factor for the 334 columns of @tab
@all.names: list of 125 components yielding allele names for each locus
@ploidy: 1
@type: codom

Optionnal contents:
@pop: - empty -
@pop.names: - empty -

@other: a list containing: x xy epid

```

```

> pop(H3N2) <- H3N2$other$epid
> dapc.flu <- dapc(H3N2, n.pca=30,n.da=10)

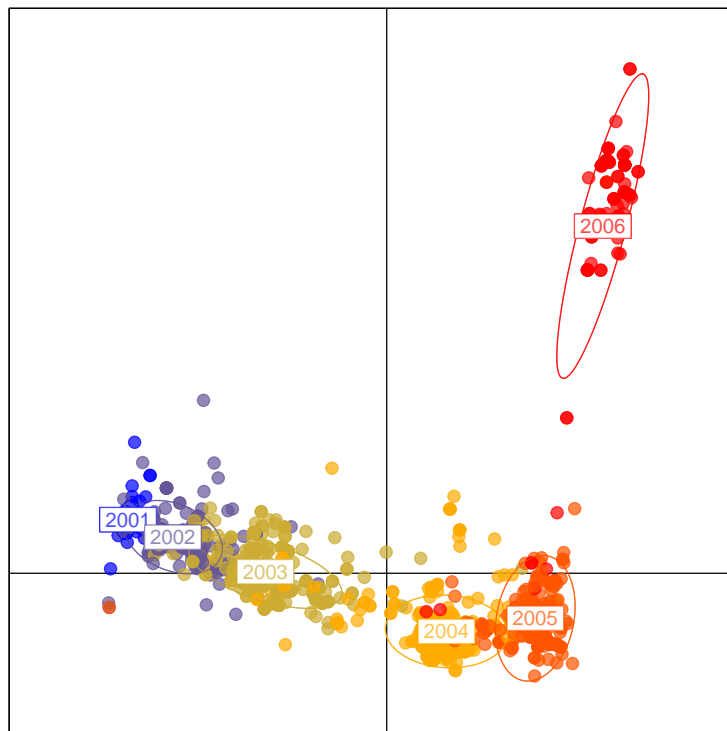
```

The first discriminant function shows the temporal evolution of the influenza virus, while the second one shows the originality of 2006 strains.

```

> myPal <- colorRampPalette(c("blue","gold","red"))
> scatter(dapc.flu, col=transp(myPal(6)), scree.da=FALSE, cell=1.5, cex=2, bg="white",cstar=0)

```

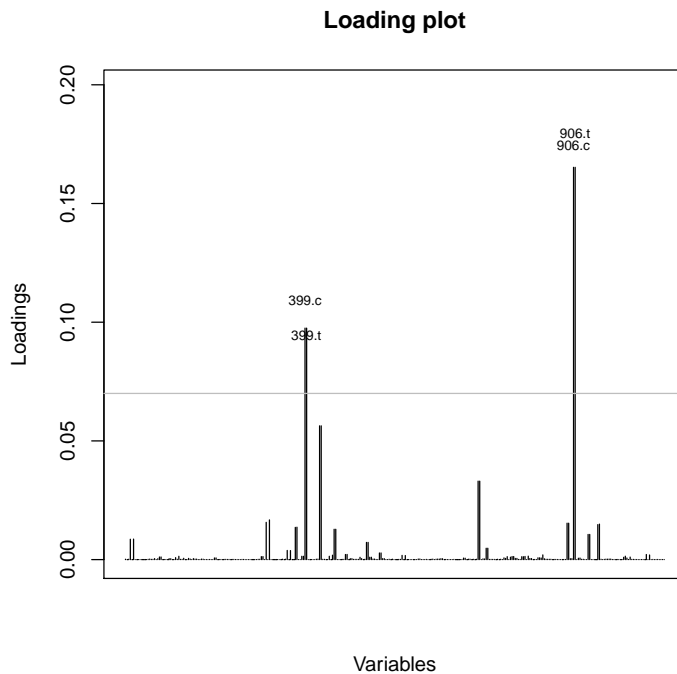


We can assess which alleles most highlight the originality of 2006 using `loadingplot`:

```

> set.seed(4)
> contrib <- loadingplot(dapc.flu$var.contr, axis=2, thres=.07, lab.jitter=1)

```



`temp` is a list invisibly returned by `loadingplot` which contains the most contributing alleles (i.e., contributions above a given threshold – argument `threshold`). In this case, SNPs 906 and 399 reflect most the temporal evolution of the virus. We can look into their allele frequencies over 2002-2006:

```
> temp <- seploc(H3N2)
> snp906 <- truenames(temp[["906"]])$tab
> snp399 <- truenames(temp[["399"]])$tab
> freq906 <- apply(snp906, 2, function(e) tapply(e, pop(H3N2), mean, na.rm=TRUE))
> freq399 <- apply(snp399, 2, function(e) tapply(e, pop(H3N2), mean, na.rm=TRUE))
> freq906
```

	906.c	906.t
2001	0.000000000	1.0000000
2002	0.000000000	1.0000000
2003	0.000000000	1.0000000
2004	0.000000000	1.0000000
2005	0.002155172	0.9978448
2006	0.616071429	0.3839286

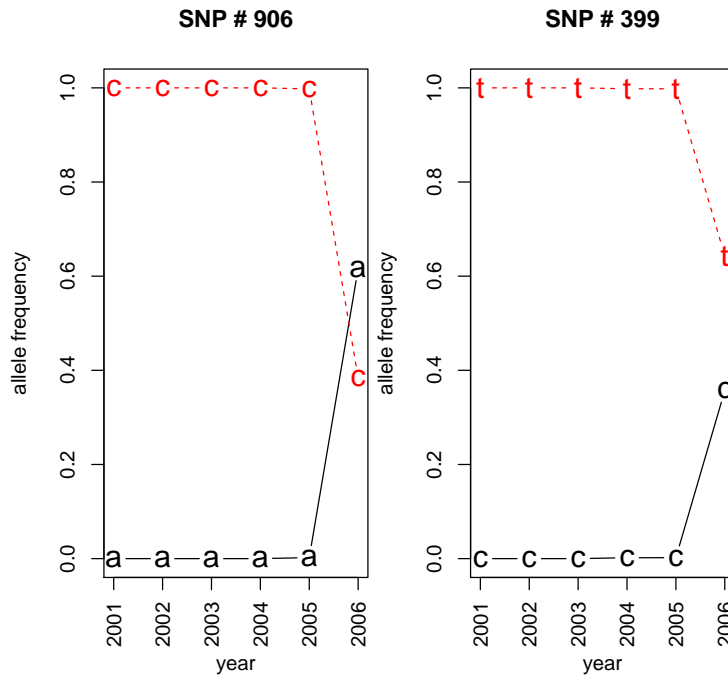
```
> freq399
```

	399.c	399.t
2001	0.000000000	1.0000000
2002	0.000000000	1.0000000
2003	0.000000000	1.0000000
2004	0.001848429	0.9981516
2005	0.002079002	0.9979210
2006	0.357142857	0.6428571

```

> par(mfrow=c(1,2), mar=c(5.1,4.1,4.1,.1),las=3)
> matplot(freq906, pch=c("a","c"), type="b",xlab="year",ylab="allele frequency", xaxt="n", cex=1.5, main="SNP # 906")
> axis(side=1, at=1:6, lab=2001:2006)
> matplot(freq399, pch=c("c","t"), type="b", xlab="year",ylab="allele frequency", xaxt="n", cex=1.5, main="SNP # 399")
> axis(side=1, at=1:6, lab=2001:2006)

```



In both cases, a new allele appeared in 2005 at a very low frequency, and reached high or even dominant frequencies a year later. Irrespective of the mechanism underlying these changes (drift or selection), this illustrates that in seasonal influenza, specific nucleotides can undergo drastic changes within only a couple of years.

### 3.5 Interpreting group memberships

Besides scatterplots of discriminant functions, group memberships of DAPC can be exploited. Note that caution should be taken when interpreting group memberships of a DAPC based on too many PCs, as there are risks of overfitting the discriminant functions (see section below). But despite this possible bias, group memberships can be used as indicators of how clear-cut genetic clusters are. Note that this is most useful for groups defined by an external criteria, i.e. defined biologically, as opposed to identified by *k*-means. It is less useful for groups identified using `find.clusters`, since we expect *k*-means to provide optimal groups for DAPC, and therefore both classifications to be mostly consistent.

Membership probabilities are based on the retained discriminant functions. They are stored in **dapc** objects in the slot **posterior**:

```
> class(dapc1$posterior)

[1] "matrix"

> dim(dapc1$posterior)

[1] 600  6

> round(head(dapc1$posterior),3)

      1 2 3      4 5 6
001 1.000 0 0 0.000 0 0
002 1.000 0 0 0.000 0 0
003 1.000 0 0 0.000 0 0
004 0.016 0 0 0.984 0 0
005 1.000 0 0 0.000 0 0
006 1.000 0 0 0.000 0 0
```

Each row corresponds to an individual, each column to a group. This information can be summarized using **summary** on the **dapc** object:

```
> summary(dapc1)

$n.dim
[1] 5

$n.pop
[1] 6

$assign.prop
[1] 0.9966667

$assign.per.pop
      1      2      3      4      5      6
1.0000000 1.0000000 1.0000000 0.9904762 1.0000000 0.9901961

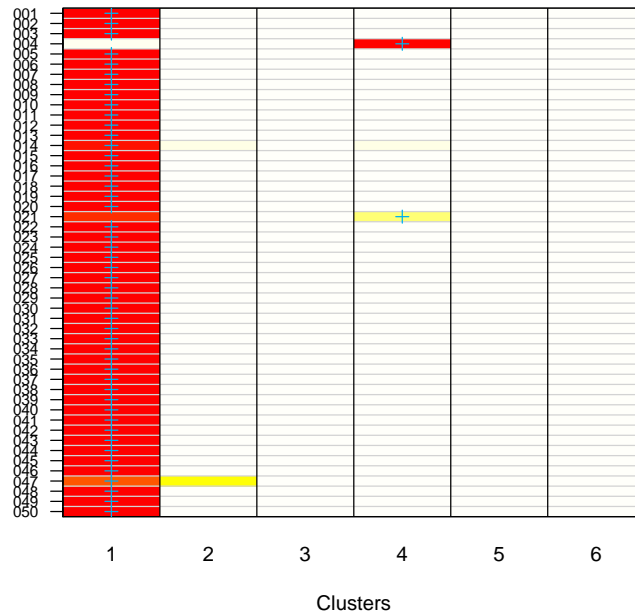
$prior.grp.size
      1 2 3 4 5 6
98 97 99 105 99 102

$post.grp.size
      1 2 3 4 5 6
99 97 99 105 99 101
```

The slot **assign.per.pop** indicates the proportions of successful reassignment (based on the discriminant functions) of individuals to their original clusters. Large values indicate clear-cut clusters, while low values suggest admixed groups.

This information can also be visualized using **assignplot** (see **?assignplot** for display options); here, we choose to represent only the first 50 individuals to make the figure readable:

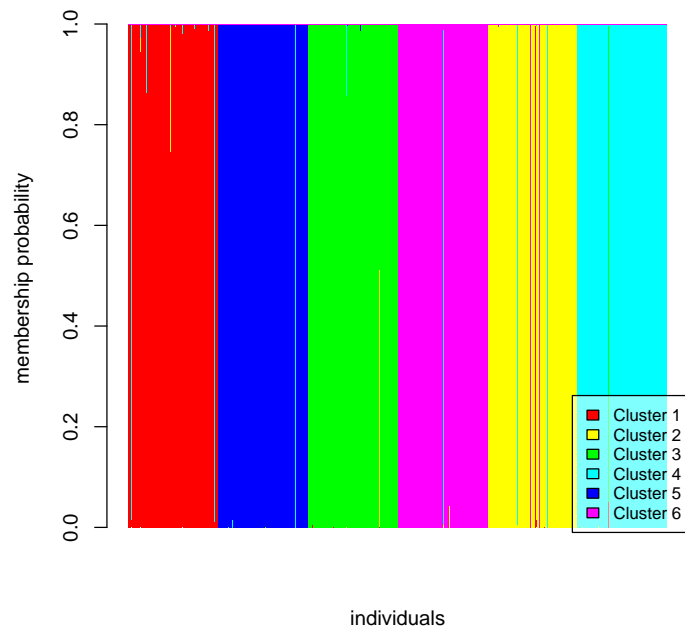
```
> assignplot(dapc1, subset=1:50)
```



This figure is the simple graphical translation of the `posterior` table above. Heat colors represent membership probabilities (red=1, white=0); blue crosses represent the prior cluster provided to DAPC. Here in most individuals, DAPC classification is consistent with the original clusters (blue crosses are on red rectangles), except for one discrepancy in individual 21, classified in group 1 while DAPC would assign it to group 3. Such figure is particularly useful when prior biological groups are used, as one may infer admixed or misclassified individuals.

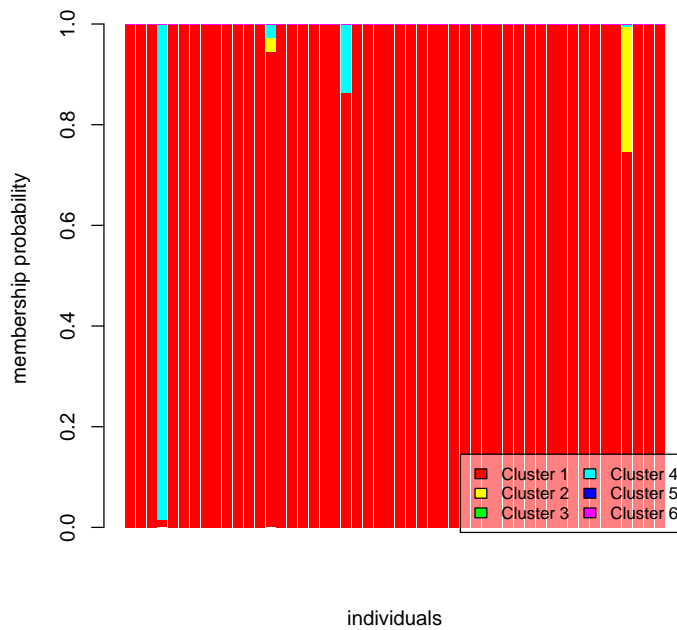
Note that this information can also be plotted in a STRUCTURE-like (!) way using `compoplot` (see `?compoplot` to customize the plot). We can plot information of all individuals to have a global picture of the clusters composition.

```
> compoplot(dapc1, posi="bottomright", txt.leg=paste("Cluster", 1:6), lab="", ncol=1, xlab="individuals")
```



We can also have a closer look at a subset of individuals; for instance, for the first 50 individuals:

```
> compoplot(dapc1, subset=1:50, posi="bottomright", txt.leg=paste("Cluster", 1:6), lab="", ncol=2, xlab="individu
```



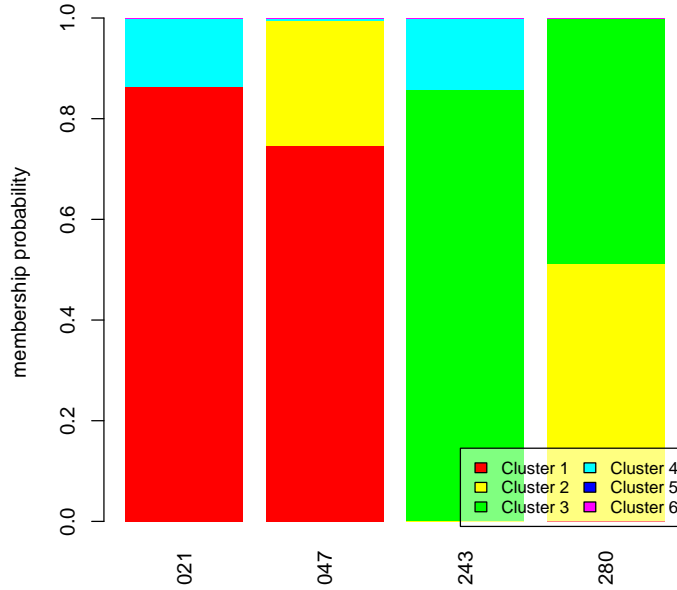
Obviously, we can use the power of R to lead our investigation further. For instance, which are the most 'admixed' individuals? Let us consider as admixed individuals having no more than 90% of probability of membership in a single cluster:

```
> temp <- which(apply(dapc1$posterior,1, function(e) all(e<0.9)))
> temp
```

```
021 047 243 280
21  47 243 280
```

```
> compoplot(dapc1, subset=temp, posi="bottomright", txt.legend=paste("Cluster", 1:6), ncol=2)
```





## 4 On the stability of group membership probabilities

### 4.1 When and why group memberships can be unreliable

In DAPC, discriminant functions are linear combinations of variables (principal components of PCA) which optimize the separation of individuals into pre-defined groups. Based on the retained discriminant functions, it is possible to derive group membership probabilities, which can be interpreted in order to assess how clear-cut or admixed the clusters are. Unfortunately, retaining too many PCs with respect to the number of individuals can lead to over-fitting the discriminant functions. In such case, discriminant functions become so "flexible" that they could discriminate almost perfectly any cluster. As a result, membership probabilities can become drastically inflated for the best-fitting cluster, resulting in apparent perfect discrimination.

This point can be illustrated using the `microbov` dataset (704 cattles of 15 breeds typed for 30 microsatellite markers). We first examine the % of successful reassignment (i.e., quality of discrimination) for different numbers of retained PCs. First, retaining 3 PCs during the dimension-reduction step, and all discriminant functions:

```

> data(microbov)
> microbov

#####
### Genind object ###
#####
- genotypes of individuals -

S4 class: genind
@call: genind(tab = truenames(microbov)$tab, pop = truenames(microbov)$pop)

@tab: 704 x 373 matrix of genotypes

@ind.names: vector of 704 individual names
@loc.names: vector of 30 locus names
@loc.nall: number of alleles per locus
@loc.fac: locus factor for the 373 columns of @tab
@all.names: list of 30 components yielding allele names for each locus
@ploidy: 2
@type: codom

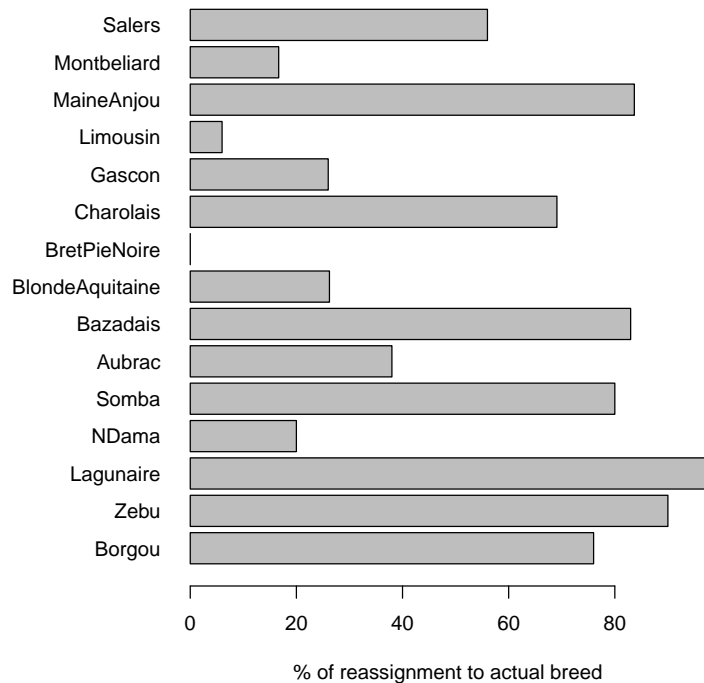
Optionnal contents:
@pop: factor giving the population of each individual
@pop.names: factor giving the population of each individual

@other: a list containing: coun breed spe

> temp <- summary(dapc(microbov, n.da=100, n.pca=3))$assign.per.pop*100

> par(mar=c(4.5,7.5,1,1))
> barplot(temp, xlab="% of reassignment to actual breed", horiz=TRUE, las=1)

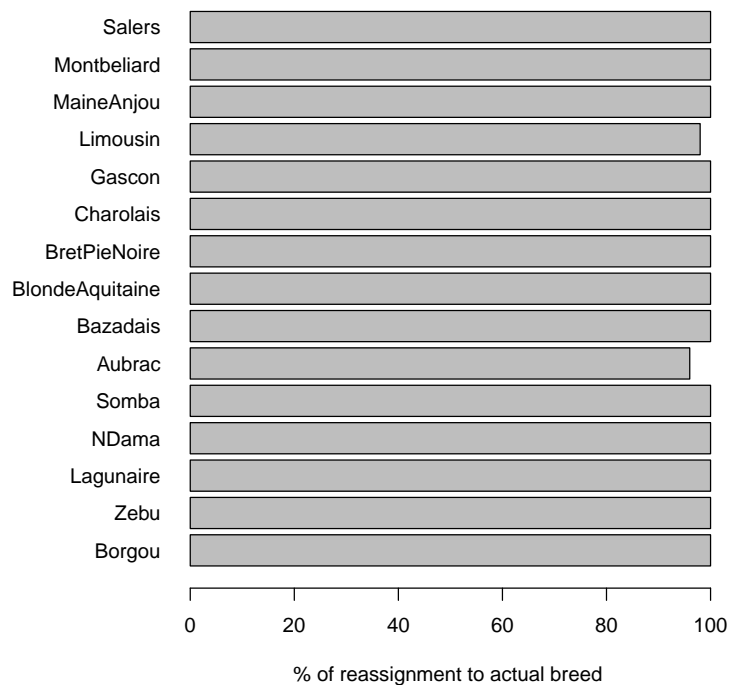
```



We can see that some breeds are well discriminated (e.g. Zebu, Lagunaire, > 90%) while others are entirely overlooked by the analysis (e.g. Bretonne Pie Noire, Limousin, <10%). This is because too much genetic information is lost when retaining only 3 PCs. We repeat the analysis, this time keeping 300 PCs:

```
> temp <- summary(dapc(microbov, n.da=100, n.pca=300))$assign.per.pop*100

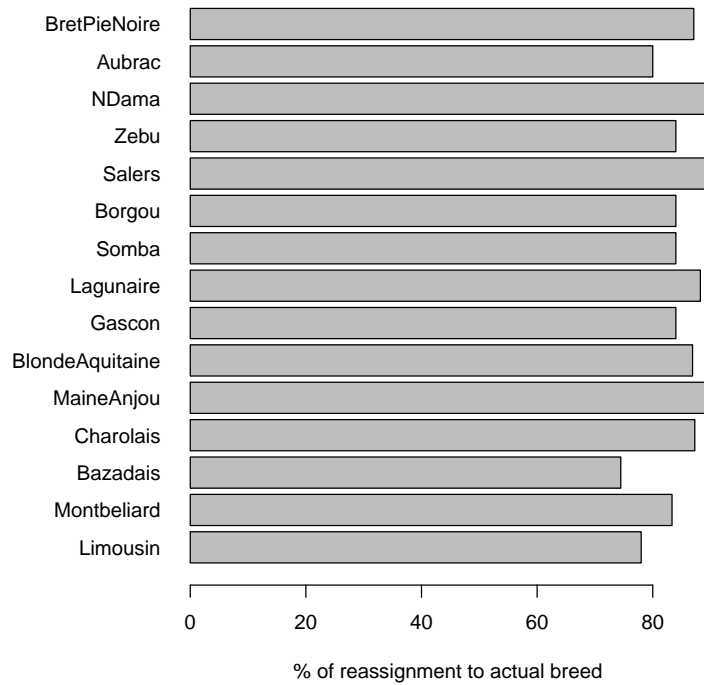
> par(mar=c(4.5,7.5,1,1))
> barplot(temp, xlab="% of reassignment to actual breed", horiz=TRUE, las=1)
```



We now obtain almost 100% of discrimination for all groups. Is this result satisfying? Actually not. The number of PCs retained is so large that discriminant functions could model any structure and virtually any set of clusters would be well discriminated. This can be illustrated by running the analysis using randomized groups:

```
> x <- microbov
> pop(x) <- sample(pop(x))
> temp <- summary(dapc(x, n.da=100, n.pca=300))$assign.per.pop*100

> par(mar=c(4.5,7.5,1,1))
> barplot(temp, xlab="% of reassignment to actual breed", horiz=TRUE, las=1)
```



Groups have been randomised, and yet we still get very good discrimination. There is therefore a trade-off between finding a space with a good power of discrimination using DAPC, and retaining too many dimensions and cause over-fitting.

## 4.2 Using the *a*-score

The trade-off between power of discrimination and over-fitting can be measured by the *a*-score, which is simply the difference between the proportion of successful reassignment of the analysis (observed discrimination) and values obtained using random groups (random discrimination). It can be seen as the proportion of successful reassignment corrected for the number of retained PCs. It is implemented by `a.score`, which relies on repeating the DAPC analysis using randomized groups, and computing *a*-scores for each group, as well as the average *a*-score:

```
> dapc2 <- dapc(microbov, n.da=100, n.pca=10)
> temp <- a.score(dapc2)
> names(temp)
```

```
[1] "tab"      "pop.score" "mean"
```

```
> temp$tab[1:5,1:5]
```

```

      Borgou Zebu Lagunaire      NDama Somba
sim.1  0.74 0.84 0.8235294 0.5666667 0.58
sim.2  0.76 0.72 0.8627451 0.5333333 0.78
sim.3  0.60 0.78 0.8627451 0.5000000 0.70
sim.4  0.64 0.74 0.8627451 0.5333333 0.70
sim.5  0.72 0.76 0.9607843 0.5000000 0.80

```

```
> temp$pop.score
```

```

      Borgou      Zebu      Lagunaire      NDama      Somba
0.6380000 0.7420000 0.8509804 0.5166667 0.7180000
Aubrac      Bazadais BlondeAquitaine BretPieNoire Charolais
0.4940000 0.8382979 0.3016393 0.4741935 0.5563636
Gascon      Limousin  MaineAnjou  Montbeliard  Salers
0.6640000 0.4280000 0.8653061 0.6733333 0.7720000

```

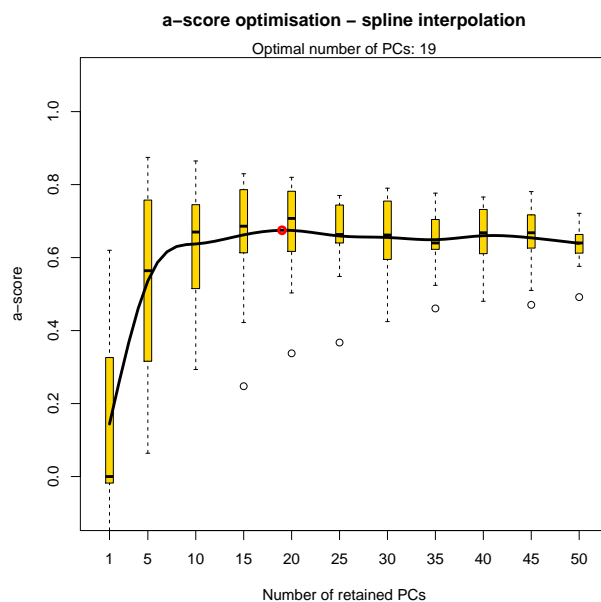
```
> temp$mean
```

```
[1] 0.6355187
```

The number of retained PCs can be chosen so as to optimize the *a*-score; this is achieved by `optim.a.score`:

```
> dapc2 <- dapc(microbov, n.da=100, n.pca=50)
```

```
> temp <- optim.a.score(dapc2)
```

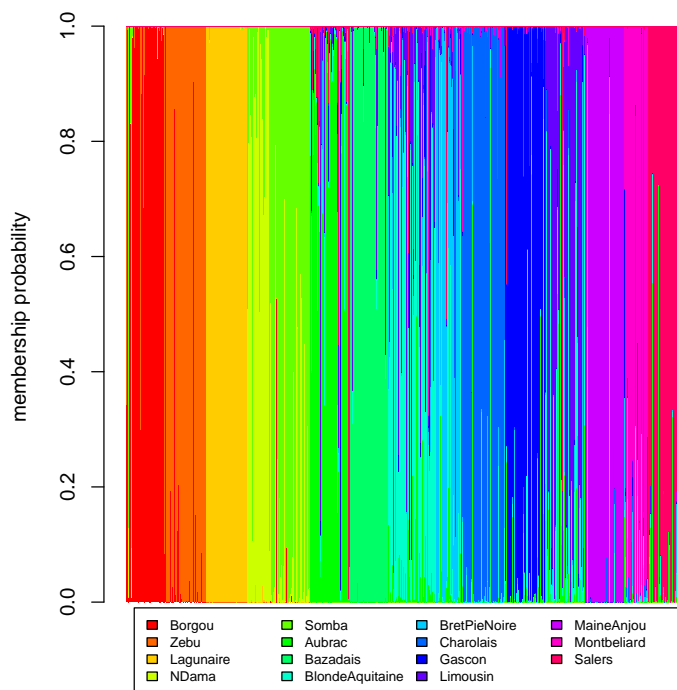


Since evaluating solutions for 1, 2, ... 100 retained PCs is unusefully computer-intensive, as a first approximation the method evaluates a few numbers of retained PCs in this range, and uses spline interpolation to approximate the optimal number of PCs to retain. Then, one can evaluate all solutions within a restrained range using the argument `n.pca`. For the `microbov` dataset, we should probably retained between 10 and 30 PCs during the dimension-reduction step.

We perform the analysis with 20 PCs retained, and then map the membership probabilities as before:

```
> dapc3 <- dapc(microbov, n.da=100, n.pca=20)
> myCol <- rainbow(15)

> par(mar=c(5.1,4.1,1.1,1.1), xpd=TRUE)
> compoplot(dapc3, lab="", posi=list(x=12,y=-.01), cleg=.7)
```



And as before, we can investigate further admixed individuals, which we arbitrarily define as those having no more than 0.5 probability of membership to any group:

```
> temp <- which(apply(dapc3$posterior,1, function(e) all(e<0.5)))
> temp
```



we may know groups for most individuals, but some individuals could be of unknown or uncertain group. In this case, we need to exclude individuals from the analysis, and then project them as supplementary individuals onto the discriminant functions. The only requirement for this operation is that supplementary individuals have been typed for the same loci as the rest of the dataset.

Technically, using supplementary individuals consists in transforming the new data using the centring and scaling of the "training data", and then using the same discriminant coefficients as for the contributing individuals to predict the position of the new individuals onto the discriminant functions.

## 5.2 In practice

We will illustrate the practice of supplementary individuals using the cattle breeds data previously analyzed (`microbov` dataset). We first split the dataset into two parts: one used for the analysis, and one used as supplementary individuals:

```
> data(microbov)
> set.seed(2)
> kept.id <- unlist(tapply(1:nInd(microbov), pop(microbov), function(e) sample(e, 20,replace=FALSE)))
> x <- microbov[kept.id]
> x.sup <- microbov[-kept.id]
> nInd(x)
```

```
[1] 300
```

```
> nInd(x.sup)
```

```
[1] 404
```

`x` is a `genind` containing the data to be analyzed; `x.sup` contains the supplementary individuals.

We perform the DAPC of `x`, and use `predict` to predict results for the supplementary individuals:

```
> dapc4 <- dapc(x,n.pca=20,n.da=15)
> pred.sup <- predict.dapc(dapc4, newdata=x.sup)
> names(pred.sup)
```

```
[1] "assign"      "posterior"   "ind.scores"
```

```
> head(pred.sup$assign)
```

```
[1] Borgou Borgou Borgou Borgou Borgou Borgou
15 Levels: Borgou Zebu Lagunaire NDama Somba Aubrac ... Salers
```

```
> pred.sup$ind.scores[1:5,1:3]
```



```

          LD1      LD2      LD3
001 -3.896992 -5.288381  0.4570651
002 -2.445063 -4.422078  0.2134797
003 -4.692576 -2.717198  0.4914203
004 -4.919515 -2.317070 -0.2390356
005 -4.718570 -0.200391 -0.9196541

```

```
> round(pred.sup$posterior[1:5, 1:5],3)
```

```

      Borgou  Zebu Lagunaire NDama Somba
001  0.612 0.388      0 0.000 0.000
002  0.983 0.017      0 0.000 0.000
003  1.000 0.000      0 0.000 0.000
004  1.000 0.000      0 0.000 0.000
005  0.688 0.000      0 0.208 0.105

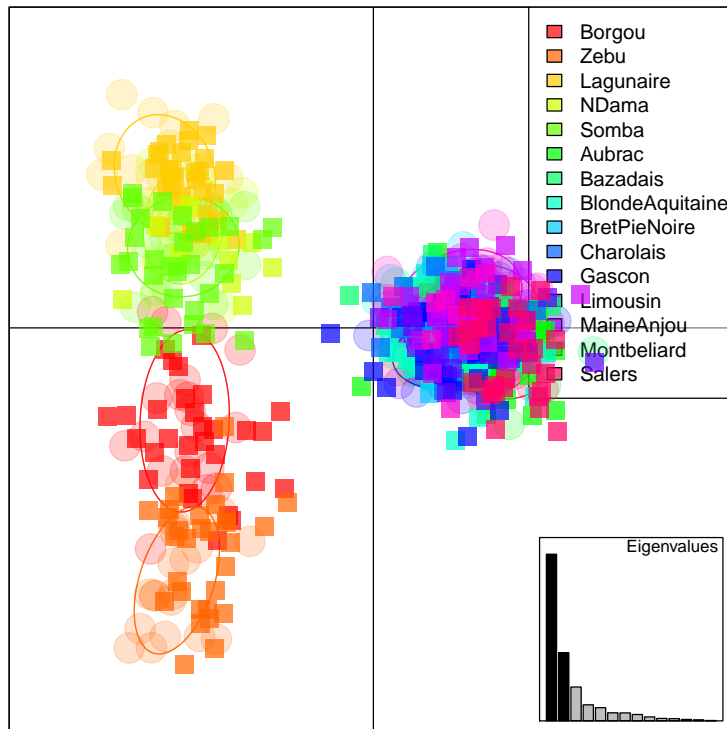
```

The list `pred.sup` contains all the predictions about the new data based on the analysis stored in `dapc4`. The slot `assign` contains the assignment of new individuals to groups; `ind.scores` contains the coordinates of the new individuals on the discriminant functions; `posterior` contains the posterior membership probabilities. We can visualize the information by different ways. First, we can represent the new individuals using a scatterplot:

```

> col <- rainbow(length(levels(pop(x))))
> col.points <- transp(col[as.integer(pop(x))],.2)
> scatter(dapc4, col=col, bg="white", scree.da=0, pch="", cstar=0, clab=0, xlim=c(-10,10), legend=TRUE)
> par(xpd=TRUE)
> points(dapc4$ind.coord[,1], dapc4$ind.coord[,2], pch=20, col=col.points, cex=5)
> col.sup <- col[as.integer(pop(x.sup))]
> points(pred.sup$ind.scores[,1], pred.sup$ind.scores[,2], pch=15, col=transp(col.sup,.7), cex=2)
> add.scatter.eig(dapc4$eig,15,1,2, posi="bottomright", inset=.02)

```



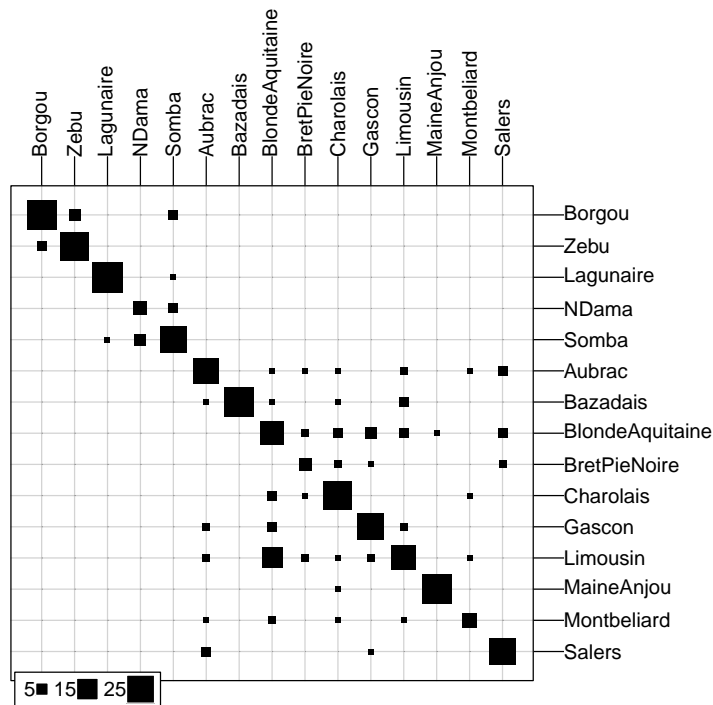
Light dots and ellipses correspond to the original analysis, while more solid squares indicate supplementary individuals. Results are fairly satisfying:

```
> mean(as.character(pred.sup$assign)==as.character(pop(x.sup)))
```

```
[1] 0.7549505
```

Around 75% of individuals have been assigned to their actual cluster. For more details about which breed was assigned to which cluster, we can display the contingency table of the actual cluster *vs* the inferred one:

```
> table.value(table(pred.sup$assign, pop(x.sup)), col.lab=levels(pop(x.sup)))
```



Columns correspond to actual clusters of the supplementary individuals, while rows correspond to inferred clusters. Overall, groups are fairly well retrieved, but we can notice that individuals of Blonde d'Aquitaine breed are poorly identified compared to other breeds.

## References

- [1] Jombart T, Devillard S and Balloux, F (2010). Discriminant analysis of principal components: a new method for the analysis of genetically structured populations. *BMC Genetics* 11: 94.

- [2] Jombart, T. (2008) adegenet: a R package for the multivariate analysis of genetic markers. *Bioinformatics* 24: 1403-1405.
- [3] R Development Core Team (2011). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. ISBN 3-900051-07-0.